Technical Report No. 43

Post-tensioned concrete floors
Design handbook

Second Edition

Report of a Concrete Society Working Party

Post-tensioned concrete floors
Design Handbook

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The Concrete Society

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SYMBOLS

A_s area of tensile reinforcement
A_c area of concrete in compression
A_s area of un-tensioned reinforcement
A_{ps} area of prestressing tendons in the tension zone
A_{sw} area of shear reinforcement in each perimeter
a drape of tendon measured at centre of profile between points of inflection
b width or effective width of the section or flange in the compression zone
b_w width of the web
$C_{Rd.c}$ coefficient
d effective depth
d' weighted average effective depth of reinforcing and bonded prestressing steel
E_c modulus of elasticity of concrete
e eccentricity of tendons
F_{bst} design bursting force
F_t tensile force to be carried by un-tensioned reinforcement
f_b bottom fibre stress
f_c compressive stress in concrete
f_{cc} compressive stress in concrete in cracked section
f_{ci} concrete cube strength at transfer
f_{ck} characteristic (cylinder) strength of concrete
f_{ct} tensile stress in concrete
f_{ctm} mean concrete tensile strength
f_{pe} design effective prestressing in tendons after all losses
f_t top fibre stress or tensile stress in concrete
f_y characteristic strength of reinforcement
$f_{ywd,ef}$ effective design strength of punching shear reinforcement
H_1 induced horizontal force at base of column 1
h depth of section
h_c effective diameter of column or column head
h_{col} height of column
I second moment of area
L span or support length
L_1 distance of column 1 from fixed support
l length of inelastic zone
l_a span for continuous slab
l_1 panel length parallel to span, measured from column centres
l_2 panel width, measured from column centres
M total out-of-balance moment
M_a applied moment due to dead and live loads
M_s moment from prestress secondary effects
$N_{E,dy}$ longitudinal force in y direction across full bay for internal columns and across control section for edge columns
$N_{E,dz}$ longitudinal force in z direction across full bay for internal columns and across control section for edge columns
n design ultimate load on full panel width between adjacent bay centre lines
P prestressing force in tendon
P_{av} average prestressing force in tendon
P_0 prestressing force at anchorage
s distance between points of inflection
s_r radial spacing of layers of shear reinforcement
u length of perimeter
$u_{out,ef}$ length of perimeter at which shear reinforcement is not required
u_{yy} total length of perimeter parallel to the Y axis
u_{zz} total length of perimeter parallel to the Z axis
V applied shear
V_{Ed} column load
V_{eff} effective applied shear (factored to take account of moment transfer effect)
V_p shear carried to column by inclined tendons
$V_{Rd,c}$ design shear resistance of concrete slab
$V_{Rd,cs}$ design shear resistance of concrete slab with shear reinforcement
$V_{Rd,max}$ maximum strut force
$v_{Rd,c}$ design shear stress resistance of concrete slab
w upward uniformly distributed load induced by tendon
x depth to neutral axis
y_p half the side of the loaded area
y_{p0} half the side of the end block
z_b bottom section modulus
z_t top section modulus

α angle between shear reinforcement and plane of slab
γ partial safety factor applied to prestressing force
δ_1 displacement of top of column 1
ε_{cc} strain in concrete at extreme fibre
ε_{LT} total long-term strain
ε_{PS} strain in prestressing strands
ε_S strain in ordinary bonded reinforcement
θ strut angle
ρ_1 $A_s/b_w d$
σ_{cp} stress due to the prestressing
σ_{cy} stress due to the prestressing parallel to the Y axis
σ_{cz} stress due to the prestressing parallel to the Z axis

1 INTRODUCTION

1.1 BACKGROUND

The use of post-tensioned concrete floors in buildings has been growing consistently in recent years. The greatest use of this type of construction has been in the USA, and in California it is the primary choice for concrete floors. Post-tensioned floors have also been used in Australia, Hong Kong, Singapore and Europe. Their use in the UK is now increasing rapidly.

Typical applications have been:

- Offices
- Car parks
- Shopping centres
- Hospitals
- Apartment buildings
- Industrial buildings
- Transfer beams
- Water-resistant roofs

These are illustrated in Figures 1–3.

The Concrete Society has published various Technical Reports on the design of post-tensioned floors[1–3]. Technical Report 43, *Post-tensioned concrete floors – Design Handbook*[4], which was published in 1994, combined the earlier reports and expanded some of the recommendations in line with current practice and the requirements of BS 8110[5]. Another important reference is the BCA report on *Post-tensioned floor construction in multi-storey buildings*[6]. The

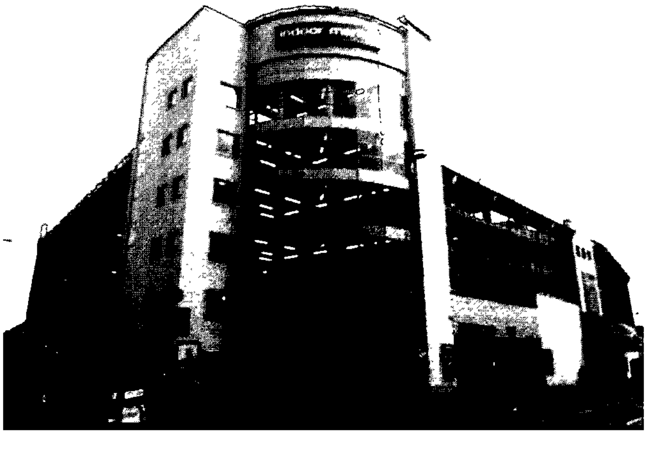


Figure 1: Bullring indoor market and multi-storey car park.

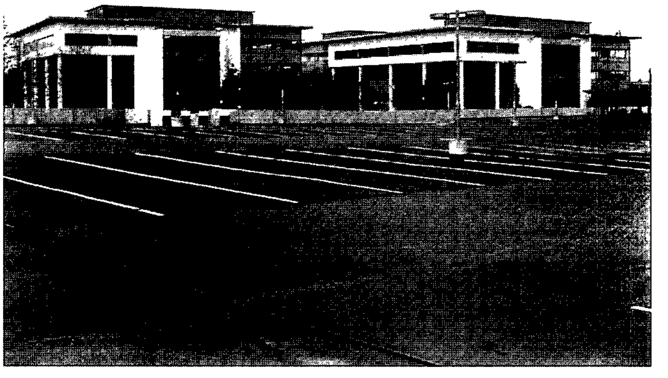


Figure 2: Office complex and car park.

Figure 3: Buchanan Street.

aim of this present Report is to further update the information in the light of developments in current practice and to align the design procedure with the recommendations of Eurocode 2[7].

This report explains the overall concept of post-tensioned concrete floor construction as well as giving detailed design recommendations. The intention is to simplify the tasks of the designer and contractor enabling them to produce effective and economic structures. Post-tensioned floors are not complex. The techniques, structural behaviour and design are simple and very similar to reinforced concrete structures. The prestressing tendons provide a suspension system within the slab and the simple arguments of the triangle of forces apply with the vertical component of the tendon force carrying part of the dead and live loading and the horizontal component reducing tensile stresses in the concrete. Examples are given in Appendix A.

The report is intended to be read in conjunction with Eurocode 2 (EC2), BS EN 1992-1-1[7] and the UK National Annex. [**Note:** At the time of preparation of this report only a draft of the National Annex was available. The reader should confirm numerical values given in Examples, etc. with the final version of the National Annex.] Those areas not covered in EC2 are described in detail in the report with references given as appropriate.

Four other Concrete Society publications give useful background information to designers of post-tensioned floors:

- Technical Report 21, *Durability of tendons in prestressed concrete*[8]
- Technical Report 23, *Partial prestressing*[9]
- Technical Report 47 (Second Edition), *Durable post-tensioned concrete bridges*[10]
- Technical Report 53, *Towards rationalising reinforcement for concrete structures*[11].

It should be noted that since the integrity of the structure depends on a relatively small number of prestressing tendons and anchorages the effect of workmanship and quality of materials can be critical. All parties involved in both design and construction should understand this. There is a specific need for extra distribution reinforcement to carry heavy point loads.

1.2 ADVANTAGES OF POST-TENSIONED FLOORS

The primary advantages of post-tensioned floors over conventional reinforced concrete in-situ floors, may be summarised as follows:

- increased clear spans
- thinner slabs
- lighter structures; reduced floor dead load
- reduced cracking and deflections
- reduced storey height
- rapid construction
- large reduction in conventional reinforcement
- better water resistance.

These advantages can result in significant savings in overall costs. There are also some situations where the height of the building is limited, in which the reduced storey height has allowed additional storeys to be constructed within the building envelope.

1.3 STRUCTURAL TYPES CONSIDERED

The report is primarily concerned with suspended floors. However, the recommendations apply equally well to foundation slabs except that since the loads are generally upward rather than downward the tendon profiles and locations of un-tensioned reinforcement are reversed.

The types of floor that can be used range from flat plates to one-way beam and slab structures. An important distinction between structural types is whether they span one-way or two-ways. This is discussed in greater detail in Section 2.2.

1.4 AMOUNT OF PRESTRESS

The amount of prestress provided is not usually sufficient to prevent tensile stresses occurring in the slab under design load conditions. The structure should therefore be considered to be partially prestressed.

The amount of prestress selected affects the un-tensioned reinforcement requirements. The greater the level of prestress, the less reinforcement is likely to be required. Unlike reinforced concrete structures, a range of acceptable designs is possible for a given geometry and loading. The optimum solution depends on the relative costs of prestressing and un-tensioned reinforcement and on the ratio of live load to dead load.

Average prestress levels usually vary from 0.7MPa to 3MPa for solid slabs and occasionally up to 6MPa for ribbed or waffle slabs. The benefits gained from prestressing reduce markedly below 0.5MPa. When the prestress exceeds 2.5MPa or the floor is very long (over 60m), the effects of restraint to slab shortening by supports may become important. If the supports are stiff a significant proportion of the prestress force goes into the supports so that the effective prestressing of the slab is reduced (see Chapter 3).

1.5 BONDED OR UNBONDED TENDON SYSTEMS

Post-tensioned floors can be constructed using either bonded or unbonded tendons. The relative merits of the two techniques are subject to debate. The following points may be made in favour of each.

1.5.1 Bonded system

For a bonded system the post-tensioned strands are installed in galvanised steel or plastic ducts that are cast into the concrete section at the required profile and form a voided path through which the strands can be installed. The ducts can be either circular- or oval-shaped and can vary in size to accommodate a varying number of steel strands within each duct. At the ends a combined anchorage casting is provided which anchors all of the strands within the duct. The anchorage transfers the force from the stressing jack into the concrete. Once the strands have been stressed the void around the strands is filled with a cementitious grout, which fully bonds the strands to the concrete. The duct and the strands contained within are collectively called a tendon.

The main features of a bonded system are summarised below.

- There is less reliance on the anchorages once the duct has been grouted.
- The full strength of the strand can be utilised at the ultimate limit state (due to strain compatibility with the concrete) and hence there is generally a lower requirement for the use of unstressed reinforcement.
- The prestressing tendons can contribute to the concrete shear capacity.
- Due to the concentrated arrangement of the strands within the ducts a high force can be applied to a small concrete section.
- Accidental damage to a tendon results in a local loss of the prestress force only and does not affect the full length of the tendon.

1.5.2 Unbonded system

In an unbonded system the individual steel strands are encapsulated in a polyurethane sheath and the voids between the sheath and the strand are filled with a rust-inhibiting grease. The sheath and grease are applied under factory conditions and the completed tendon is electronically tested to ensure that the process has been carried out successfully. The individual tendons are anchored at each end with anchorage castings. The tendons are cast into the concrete section and are jacked to apply the required prestress force once the concrete has achieved the required strength.

The main features of an unbonded system are summarised below.

- The tendon can be prefabricated off site.
- The installation process on site can be quicker due to prefabrication and the reduced site operations.
- The smaller tendon diameter and reduced cover requirements allow the eccentricity from the neutral axis to be increased thus resulting in a lower force requirement.
- The tendons are flexible and can be curved easily in the horizontal direction to accommodate curved buildings or divert around openings in the slab.
- The force loss due to friction is lower than for bonded tendons due to the action of the grease.
- The force in an unbonded tendon does not increase significantly above that of the prestressing load.
- The ultimate flexural capacity of sections with unbonded tendons is less than that with bonded tendons but much greater deflections will take place before yielding of the steel.
- Tendons can be replaced (usually with a smaller diameter).
- A broken tendon causes prestress to be lost for the full length of that tendon.
- Careful attention is required in design to ensure against progressive collapse.

1.6 ANALYTICAL TECHNIQUES

The design process is described in Chapter 5. The main analytical techniques used for prestressed floors include the 'equivalent frame', grillage and finite element methods. In addition to standard plane frame programs, there are available a number of programs, specifically written for the design of prestressed structures. These programs reduce the design time but are not essential for the design of post-tensioned floors. Recently more use has been made of proprietary grillage and finite element analysis and design packages.

2 STRUCTURAL BEHAVIOUR

2.1 EFFECTS OF PRESTRESS

The primary effects of prestress are axial pre-compression of the floor and an upward load within the span that balances part of the downward dead and live loads. This transverse effect carries the load directly to the supports. For the remaining load the structure will have an enhanced resistance to shear, punching and torsion due to the compressive stresses from the axial effect. In a reinforced concrete floor, tensile cracking of the concrete is a necessary accompaniment to the generation of economic stress levels in the reinforcement. In post-tensioned floors both the pre-compression and the upward load in the span act to reduce the tensile stresses in the concrete. This reduces deflection and cracking under service conditions.

However, the level of prestress is not usually enough to prevent all tensile cracking under full design live loading at Serviceability Limit State. Under reduced live load much of the cracking will not be visible.

Flexural cracking is initiated on the top surface of the slab at column faces and can occur at load levels in the serviceability range. While these and early radial cracks remain small, they are unlikely to affect the performance of the slab. Compression due to prestress delays the formation of cracks, but it is less efficient in controlling cracking, once it has occurred, than un-tensioned reinforcement placed in the top of floors, immediately adjacent to, and above the column.

The act of prestressing causes the floor to bend, shorten, deflect and rotate. If any of these effects are restrained, secondary effects of prestress are set up. These effects should always be considered. It should be noted that if there are stiff restraints in the layout of the building (e.g. two core structures at each end of the building) much of the P/A from the applied prestress will be lost (see Section 3.1).

Secondary effects are discussed in more detail in Section 5.6 and the calculation of these effects is described in Appendix D.

2.2 ONE-WAY AND TWO-WAY SPANNING FLOORS

There are several different types of post-tensioned floor. Some of the more common layouts are given in Figures 4–7. An important distinction between types of floors is whether they are one-way or two-way spanning structures. In this design handbook the term 'flat slab' means two-way spanning slabs supported on discrete columns.

One-way floors carry the applied loading primarily in one direction and are treated as beams or plane frames. On the other hand, two-way spanning floors have the ability to sustain the applied loading in two directions. However, for a structure to be considered to be two-way spanning it must meet several criteria. These criteria are discussed in Section 2.4.

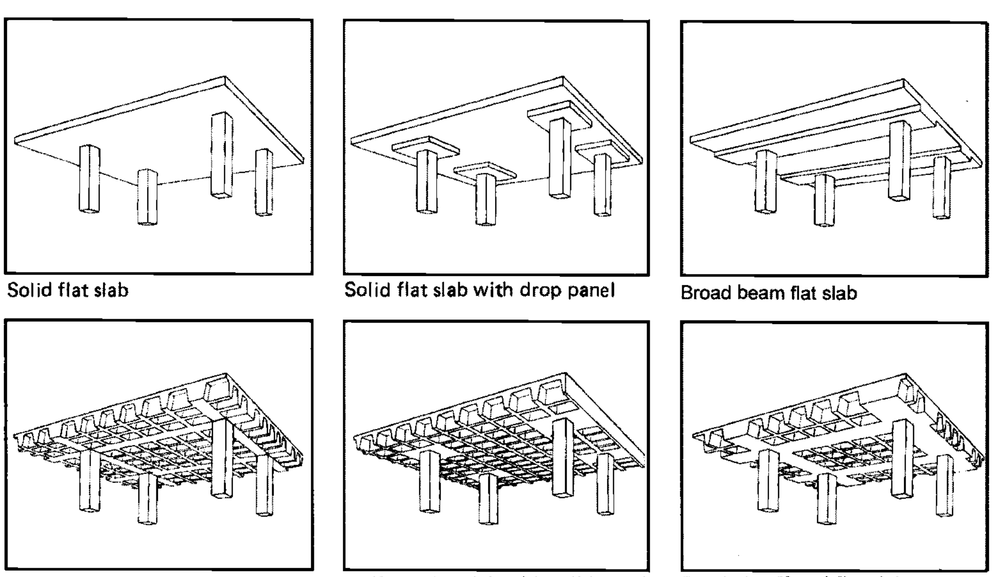
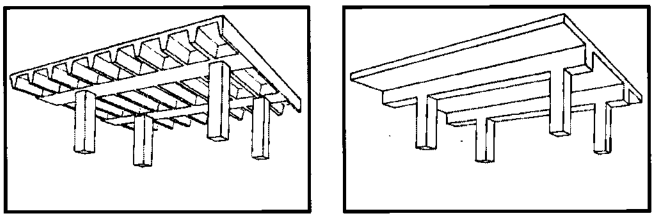


Figure 4: Typical flat slabs. See Section 2.4 for limiting criteria of two-way action.



Ribbed slab **Beam and slab**

Figure 5: Typical one-way spanning floors.

Figure 6: Post-tensioned ribbed slab.

Figure 7: Bullring multi-storey car park.

2.3 FLEXURE IN ONE-WAY SPANNING FLOORS

Prestressed one-way spanning floors are usually designed assuming some cracking occurs. Although cracking is permitted, it is assumed in analysis that the concrete section is uncracked and the tensile stress is limited to $f_{ct,eff}$ (see Eurocode 2, Clause 7.1 (2)) at Serviceability Limit State. In such situations the deflection may be predicted using gross (concrete and reinforcement) section properties.

In other cases, where the tensile stress is not limited to $f_{ct,eff}$, calculation of deflections should be based on the moment–curvature relationship for cracked sections.

2.4 FLEXURE IN FLAT SLABS

2.4.1 Flat slab criteria

For a prestressed floor, without primary reinforcement, to be considered as a flat slab the following criteria apply:

- Pre-compression is normally applied in two orthogonal directions:
  Such a floor with no, or moderate, crack formation performs as a homogeneous elastic plate with its inherent two-way behaviour. The actual tendon location at a given point in a floor system is not critical to the floor's two-way behaviour since axial compression, which is the main component of prestressing, is commonly applied to the floor at its perimeter.

  The pre-compression at the edges of the slab is concentrated behind the anchorages, and spreads into the floor with increasing distance from the edge. This is true for floors of uniform thickness as well as floors with beams in the direction of pre-compression. Floors with banded post-tensioning and floors with wide shallow beams also qualify for two-way action at regions away from the free edges where pre-compression is attained in both directions.

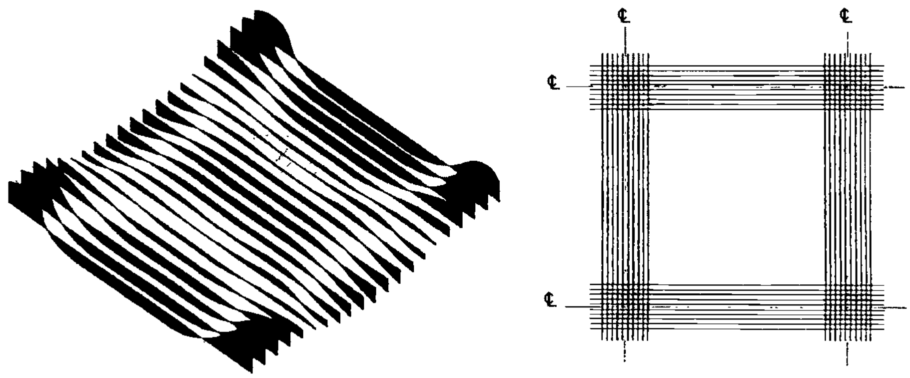
  Past experience shows that for the pre-compression to be effective it should be at least 0.7MPa in each direction.

  Flat slab behaviour is, of course, possible with pre-compression applied in one direction only. However in that situation it must be fully reinforced in the direction not prestressed. Particular care should be taken to avoid overstressing during construction (e.g. striking of formwork).

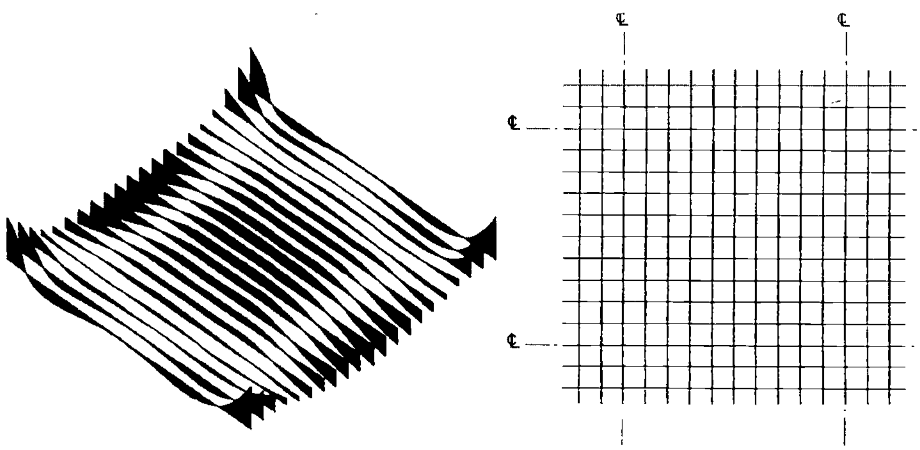
- Aspect ratio (length to width) of any panel should not be greater than 2.0:
  This applies to solid flat slabs, supported on orthogonal rows of columns. For aspect ratios greater than 2.0 the middle section will tend to act as a one-way spanning slab.

- Stiffness ratios in two directions:
  The ratio of the stiffness of the slab in two orthogonal directions should not be disproportionate. This is more likely to occur with non-uniform cross-sections such as ribs. For square panels this ratio should not exceed 4.0, otherwise the slab is more likely to behave as one-way spanning.

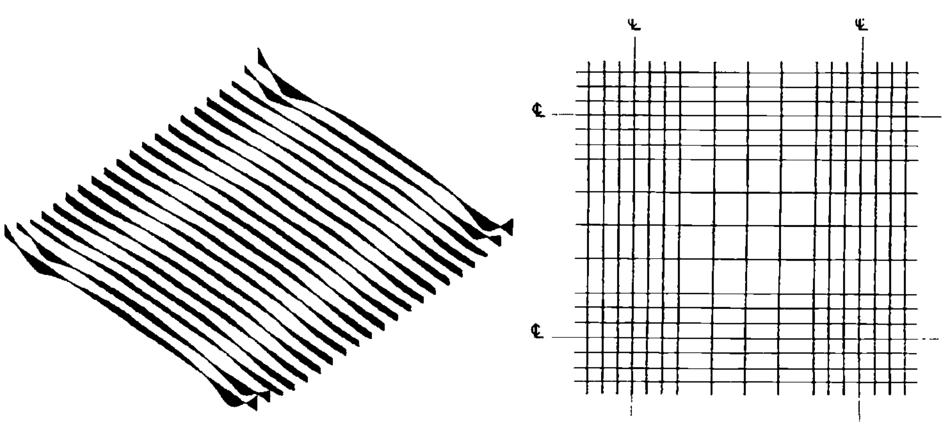
- Number of panels:
  Where the number of panels is less than three in either direction the use of the empirical coefficient method, for obtaining moments and forces, is not applicable. In such situations a more rigorous analysis should be carried out (see Section 5.7).



a) Fully banded tendons (reinforced between bands)



b) Uniformly distributed tendons



c) 50% banded plus 50% evenly distributed tendons

Figure 8: Bending moment surfaces for different arrangements of tendons.

2.4.2 Post-tensioned flat slab behaviour

Tests and applications have demonstrated that a post-tensioned flat slab behaves as a flat plate almost regardless of tendon arrangement (see Figure 8). The effects of the tendons are, of course, critical to the behaviour as they exert loads on the slab as well as provide reinforcement. The tendons exert vertical loads on the slab known as equivalent loads (see Section 5.4), and these loads may be considered like any other dead or live load. The objective is to apply prestress to reduce or reverse the effects of gravity in a uniform manner. Although the shape of the equivalent bending moment diagram from prestress is not the same as that from uniformly distributed loading such as self-weight, it is possible, with careful placing of the prestressing tendons, to achieve a reasonable match as shown in Figure 8. It should be noted that this will cause the peaks of resulting moments to appear in odd places.

The balanced load provided by the tendons in each direction is equal to the dead load. Figure 8c gives the most uniform distribution of moments. However this does not provide a practical layout of tendons as it requires knitting them over the column.

The distribution of moments for a flat plate, shown in Figures 9 and 10, reveals that hogging moments across a panel are sharply peaked in the immediate vicinity of the column and that the moment at the column face is several times the moment midway between columns. It should be noted that the permissible stresses given in Table 4 of Section 5.8.1 are average stresses for the full panel assuming an equivalent frame analysis. They are lower than those for one-way floors to allow for this non-uniform distribution of moments across the panel. The permissible stresses given in Table 2b assume a grillage or finite element (FE) analysis.

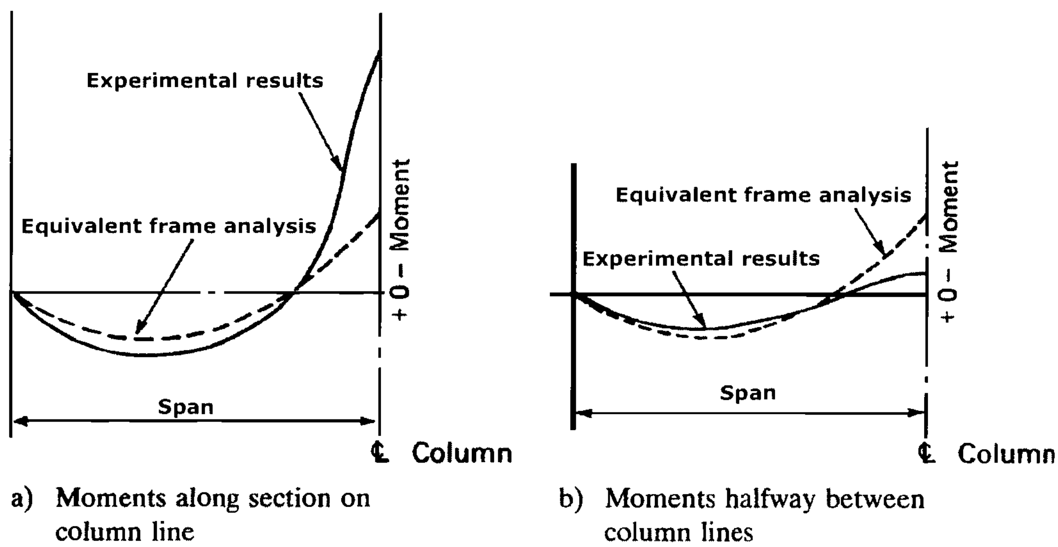


Figure 9: Applied load bending moments in a solid flat slab.

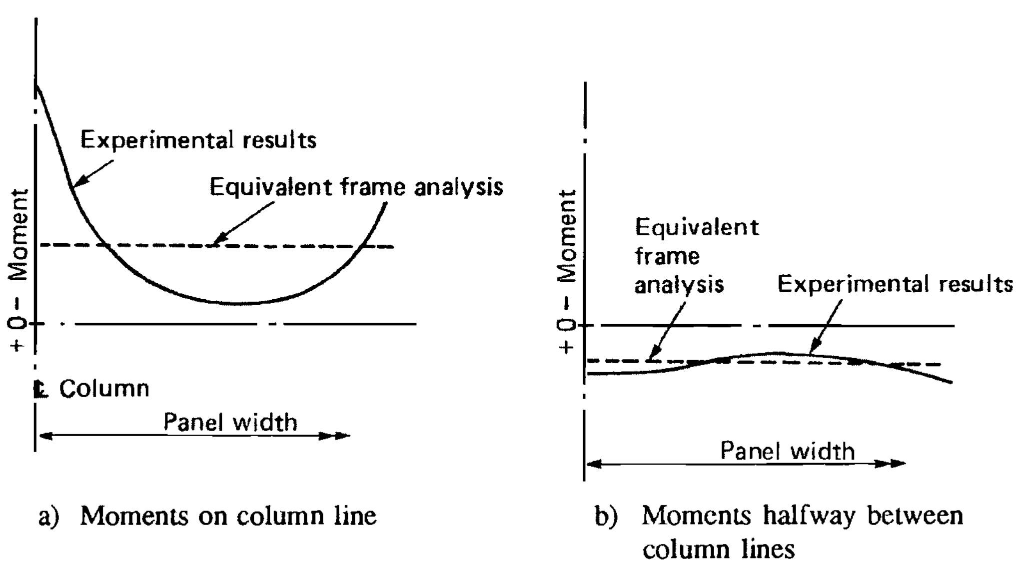


Figure 10: Distribution of applied load bending moments across the width of a panel in a solid flat slab.

In contrast the sagging moments across the slab in mid-span regions are almost uniformly distributed across the panel width as shown in Figure 10b.

It is helpful to the understanding of post-tensioned flat slabs to forget the arbitrary column strip, middle strip and moment percentage tables which have long been familiar to the designer of reinforced concrete floors. Instead, the mechanics of the action of the tendons will be examined first.

The 'load balancing' approach is an even more powerful tool for examining the behaviour of two-way spanning systems than it is for one-way spanning members. By the balanced load approach, attention is focused on the loads exerted on the floor by the tendons, perpendicular to the plane of the floor. As for one-way floors, this typically means a uniform load exerted upward along the major portion of the central length of a tendon span, and statically equivalent downward load exerted over the short length of reverse curvature. In order to apply an essentially uniform upward load over the entire floor panel these tendons should be uniformly distributed, and the downward loads from the tendons should react against another structural element. The additional element could be a beam or wall in the case of one-way floors, or columns in a two-way system. However, a look at a plan view of a flat slab (see Figure 11) reveals that columns provide an upward reaction for only a very small area. Thus, to maintain static rationality a second set of tendons perpendicular to the above tendons must provide an upward load to resist the downward load from the first set. Remembering that the downward load of the uniformly distributed tendons occurs over a relatively narrow width under the reverse curvatures and that the only available exterior reaction, the column, is also relatively narrow, it indicates that the second set of tendons should be in narrow strips or bands passing over the columns.

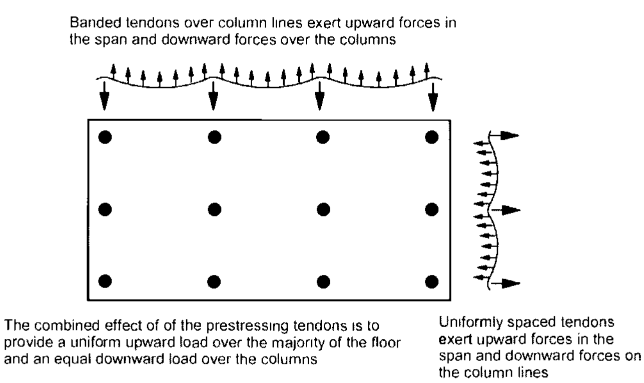


Figure 11: Load balancing with prestress tendons for regular column layouts.

Methods of accomplishing this two-part tendon system to obtain a nearly uniform upward load may be obtained by a combination of spreading the tendons uniformly across the width of the slab and/or banding them over the column lines. Figures 12 and 13 show two examples. The choice of the detailed distribution is not critical, as can be seen from Figure 8, provided that sufficient tendons pass through the column zone to give adequate protection against punching shear and progressive collapse.

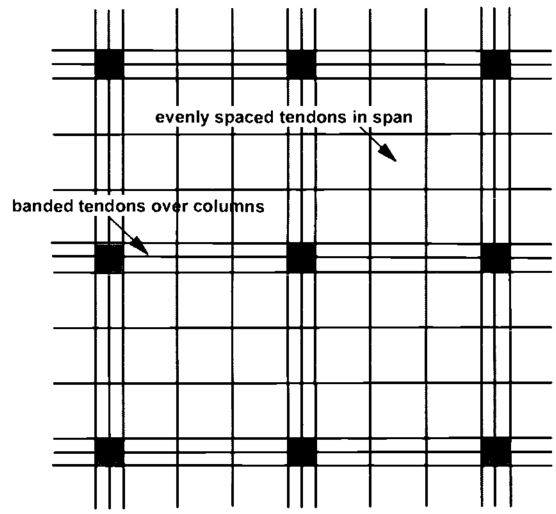


Figure 12: Tendons geometrically banded in each direction.

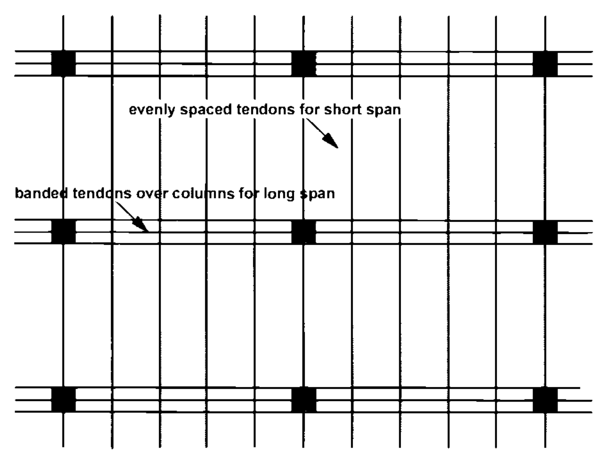


Figure 13: Tendons fully banded in one direction and uniformly distributed in the other direction.

The use of finite element or grillage methods shows that the distribution of bending moments is characterised by hogging moments which are sharply peaked in the immediate vicinity of the columns. The magnitude of the hogging moments locally to the column face can be several times that of the sagging moments in the mid-span zones.

A typical distribution of bending stresses for a uniformly loaded regular layout is illustrated in Figure 14.

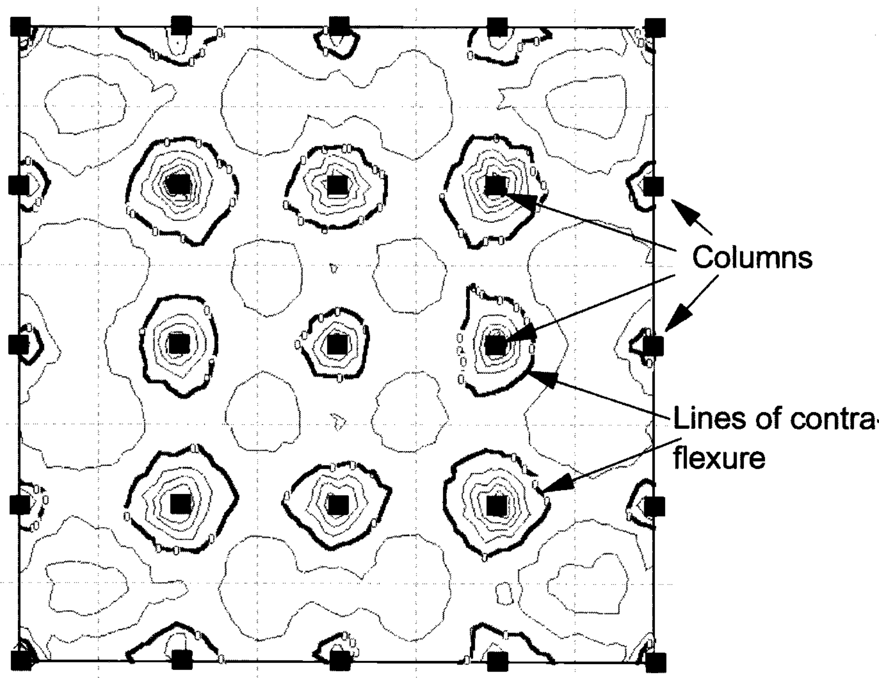


Figure 14: Typical distribution of bending stress for a uniformly loaded regular layout.

2.5 SHEAR

The method for calculating shear is given in EC2, Clause 6.2 and for punching shear in Clause 6.4. Further advice for the design of punching shear reinforcement in post-tensioned flat slabs is given in Section 5.9 of this Report.

3 STRUCTURAL FORM

3.1 PLAN LAYOUT

Current experience in many countries indicates a minimum span of approximately 7m to make prestressing viable in a floor. However, examples are known in which prestressed floors have been competitive where shorter spans have been used for architectural reasons, but prestressing was then only made viable by choosing the right slab form. In general the ideal situation is, of course, to 'think prestressing' from the initial concept of the building and to choose suitably longer spans.

In choosing column and wall layouts and spans for a prestressed floor, several possibilities may be considered to optimise the design, which include:

a) Reduce the length of the end spans or, if the architectural considerations permit, inset the columns from the building perimeter to provide small cantilevers (see Figure 15). Consequently, end span bending moments will be reduced and a more equable bending moment configuration obtained.

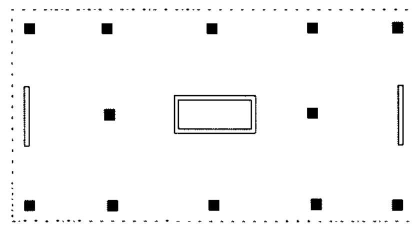
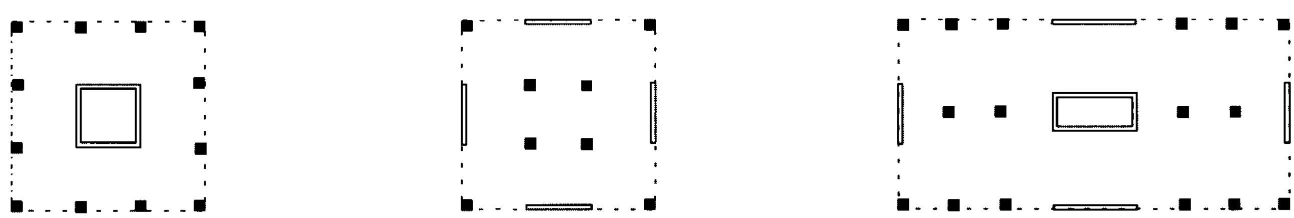


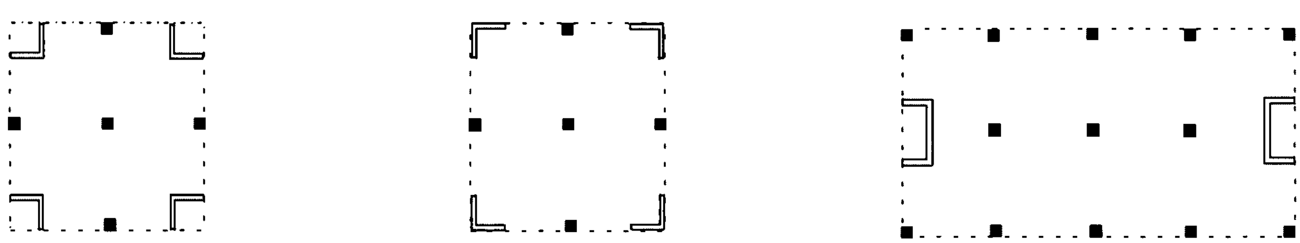
Figure 15: Typical floor layout to maximise prestressing effects.

b) Reduce, if necessary, the stiffness of the columns or walls in the direction of the prestressing to minimise the prestress lost and resulting cracking in overcoming the restraint offered to floor shortening (see Section 3.3). Figure 16 shows some typical floor layouts. Favourable layouts (see Figure 16a) allow the floors to shorten towards the stiff walls. Unfavourable layouts (see Figure 16b) restrain the floors from shortening.

c) Where span lengths vary, adjust the tendon profiles and the number of tendons to provide the uplift required for each span. Generally this will be a similar percentage of the dead load for each span.



a) Favourable layout of restraining walls.



b) Unfavourable layout of restraining walls.

Figure 16: Layout of shear walls to reduce loss of prestress and cracking effects.

Once the layout of columns and walls has been determined, the next consideration is the type of floor to be used. This again is determined by a number of factors such as span lengths, magnitude of loading, architectural form and use of the building, special requirements such as services, location of building and the cost of materials available.

3.2 FLOOR THICKNESS AND TYPES

The slab thickness must meet two primary functional requirements – structural strength and deflection. Vibration should also be considered where there are only a few panels. The selection of thickness or type (e.g. plate without drops, plate with drops, coffered or waffle, ribbed or even beam and slab) is also influenced by concrete strength and loading. There are likely to be several alternative solutions to the same problem and a preliminary costing exercise may be necessary in order to choose the most economical.

The information given in Figures 17–19 will assist the designer to make a preliminary choice of floor section. Figure 17 (derived from Table 1) gives typical imposed load capacities for a variety of flat slabs and one-way floors over a range of span/depth ratios. These figures are based on past experience. Figure 17 is appropriate for all types of prestressed floor. Figures 18 and 19 are only appropriate for flat slabs but Figure 18 is not appropriate for coffered slabs that do not have a solid section over the column.

At this stage it should be noted that the superimposed load used in Figures 17–19 consists of all loading (dead and live) bar the self-weight of the section. The calculation methods used for obtaining the graphs in Figures 19 and 20 are described in Appendix F.

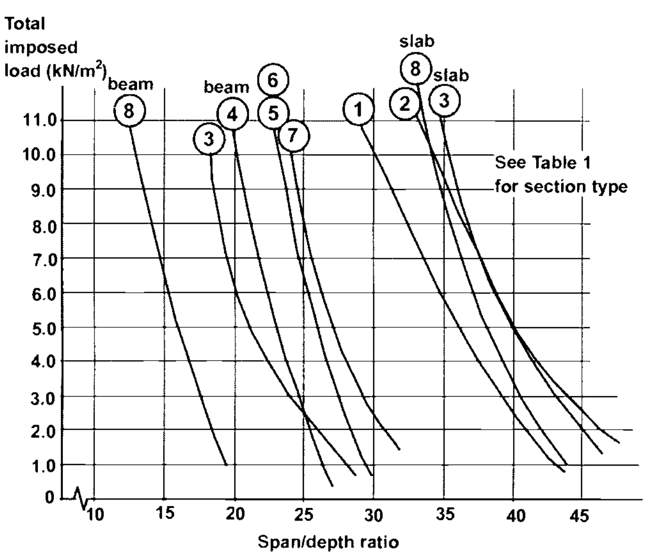
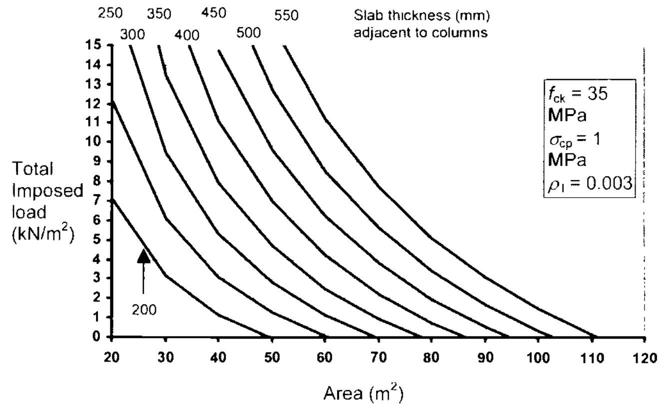
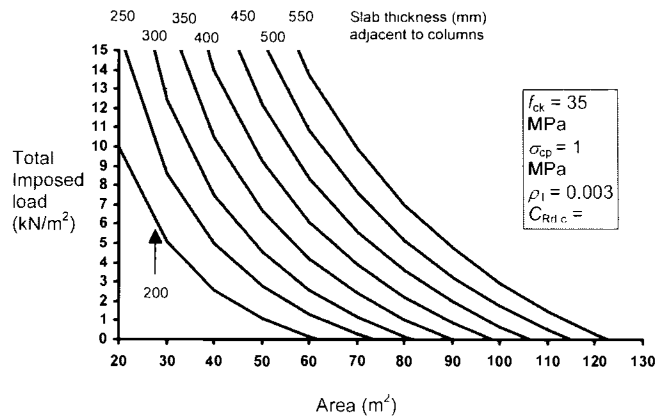


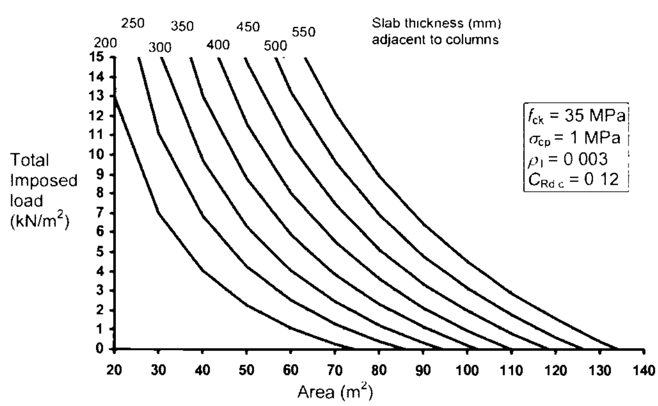
Figure 17: Preliminary selection of floor thickness for multi-span floors.



a) Column size including head = 300 mm



b) Column size including head = 500 mm



c) Column size including head = 700 mm

Figure 18: Preliminary shear check for slab thickness at internal column.

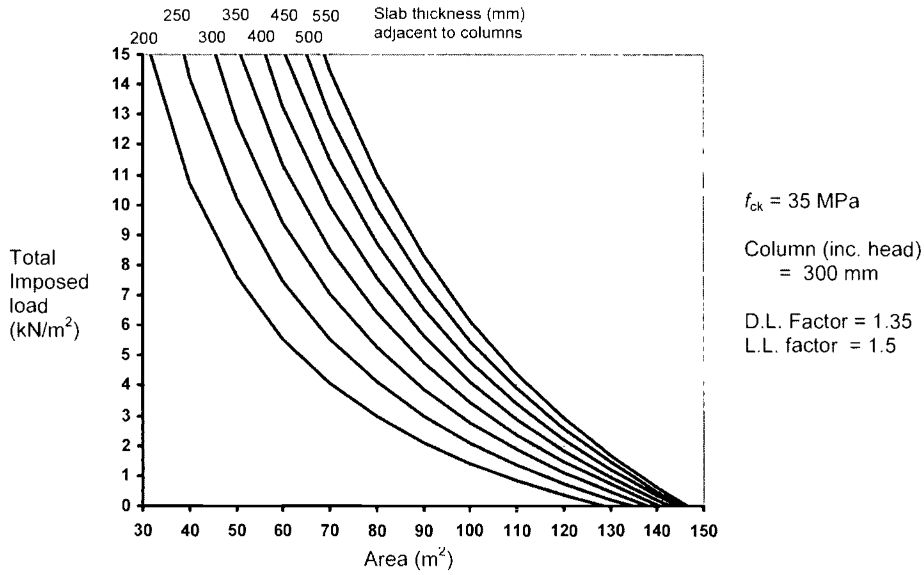


Figure 19: Ultimate shear check for flat slab at face of internal column.

Notes to Figure 19:

1. The graph has been derived for slabs with 300 × 300mm supporting columns. For column sizes larger than 300mm the area may be multiplied by the factor (column perimeter / 1200).
2. For concrete strengths other than f_{ck} = 35MPa the area should be multiplied by the factor $[(0.17f_{ck} - 0.00068f_{ck}^2) / 5.12]$.
3. The value $h - d$ is assumed to be 35mm.
4. The equivalent overall load factor assumed is 1.42 (Characteristic Dead Load + Characteristic Total Imposed Load). This factor is dependent on the dead/live load ratio.
5. The value of V_{eff} / V is assumed to be 1.15.
6. These curves do not take account of elastic distribution effects (see Section 5).

Flat slabs tend to exceed punching shear limits around columns, and often need additional shear reinforcement at these locations. The graphs in Figure 18 provide a preliminary assessment as to whether shear reinforcement is needed for the section types 1, 2, 3, 5 and 6 (all flat slabs) in Table 1. As the shear capacity of a slab is dependent on the dimensions of the supporting columns or column heads, each graph has been derived using different column dimensions.

In addition, the shear capacity at the face of the column should be checked. This can be done using the graph in Figure 19.

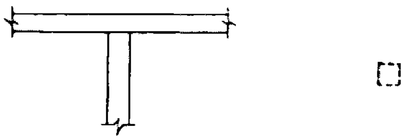
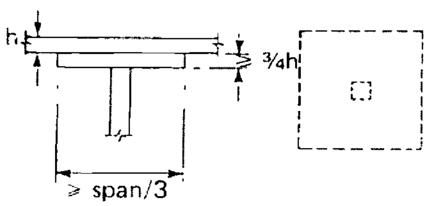
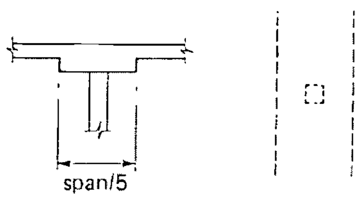
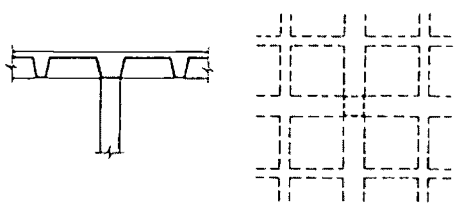
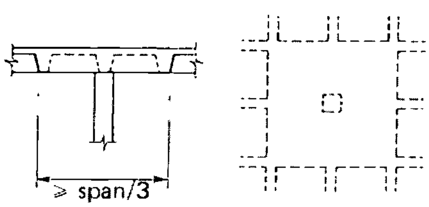
The following procedure should be followed when using Table 1 and Figures 17–19 to obtain a slab section.

a) Knowing the span and imposed loading requirements, Figure 17 or Table 1 can be used to choose a suitable span/depth ratio for the section type being considered. Table 1 also provides a simple check for vibration effects for normal uses.
b) If section type 1, 2, 3, 5 or 6 has been chosen, check the shear capacity of the section, using one of the graphs in Figure 18 (depending on what size of column has been decided upon). Obtain the imposed load capacity for the chosen slab section. If this exceeds the imposed load, then shear reinforcement is unlikely to be necessary. If it does not, then reinforcement will be required. If the difference is very large, then an increase in section depth or column size should be considered.
c) Check the shear capacity at the face of the column using the graph in Figure 19. If the imposed load capacity is exceeded, increase the slab depth and check again.

It should be noted that Table 1 and Figure 17 are applicable for multi-span floors only. For single-span floors the depth should be increased by approximately 15%. Figures 18 and 19 are applicable for both floor types and have been derived using an average load factor of 1.5 (see Appendix F).

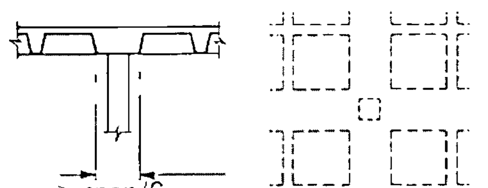
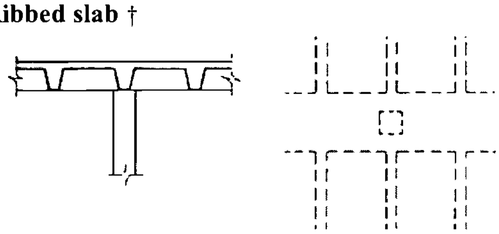
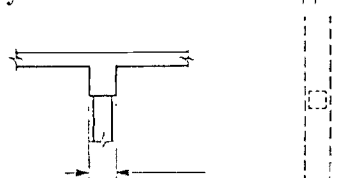
Figures 18 and 19 are set for internal columns. They may be used for external columns provided that the loaded area is multiplied by 2 × 1.4/1.15 = 2.45 for edge and 4 × 1.5/1.15 = 5.25 (applying the simplified values of b from Eurocode 2, Clause 6.4.3 (5)) for the corner columns. This assumes that the edge of the slab extends to at least the centre line of the column.

Table 1: Typical span/depth ratios for a variety of section types for multi-span floors.

Section type	Total imposed load (kN/m)	Span/depth ratios $6 \text{ m} \leq L \leq 13 \text{ m}$ (kN/m)	Additional requirements for vibration
1. Solid flat slab 	2.5 5.0 10.0	40 36 30	**A**
2. Solid flat slabwith drop panel 	2.5 5.0 10.0	44 40 36	**A**
3. Banded flat slab 	2.5 5.0 10.0	Slab Beam 45 25 40 22 35 18	**A**
4. Coffered flat slab 	2.5 5.0 10.0	25 23 20	**B**
5. Coffered flat slabwith solid panels 	2.5 5.0 10.0	28 26 23	**B**

See notes on following page.

Table 1: Typical span/depth ratios for a variety of section types for multi-span floors (continued).

Section type	Total imposed load (kN/m)	Span/depth ratios 6 m ≤ L ≤ 13 m (kN/m)		Additional requirements for vibration
6. Coffered slab with band beam				
 > span/6	2.5	28		B
	5.0	26		
	10.0	23		
7. Ribbed slab †				
	2.5	30		B
	5.0	27		
	10.0	24		
8. One-way slab with narrow beam ††		**Slab**	**Beam**	
 span/15	2.5	42	18	A
	5.0	38	16	
	10.0	34	13	

Notes:

1. Vibration. The following additional check should be made for normal office conditions if no further vibration checks are carried out (otherwise refer to Appendix G):
   A Either the floor has at least four panels and is at least 250mm thick or the floor has at least eight panels and is at least 200mm thick.
   B Either thc floor has at least four panels and is at least 400mm thick or the floor has at least eight panels and is at least 300mm thick.
2. All panels assumed to be square.
3. Span/depth ratios not affected by column head.
4. †It may be possible that prestressed tendons will not be required in the banded sections and that un-tensioned reinforcement will suffice in the ribs, or vice versa.
   ††The values of span/depth ratio can vary according to the width of the beam.

3.3 EFFECT OF RESTRAINT TO FLOOR SHORTENING

A post-tensioned floor must be allowed to shorten to enable the prestress to be applied to the floor. Shortening occurs because of:

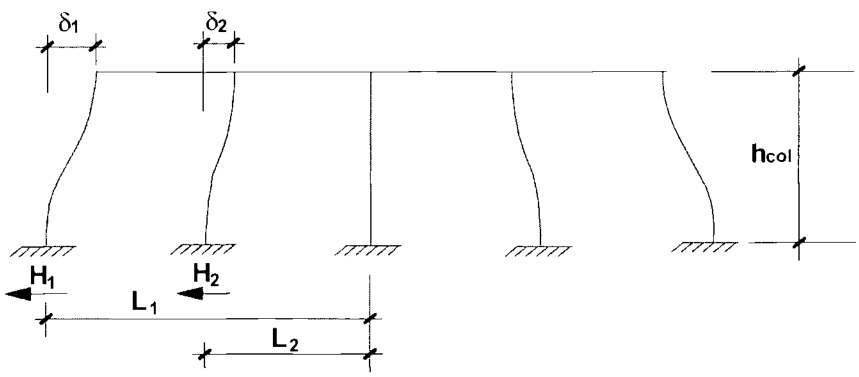
a) Shrinkage from early thermal effects (see Appendix H)
b) Elastic shortening due to the prestress force
c) Creep (including shortening due to the prestress force)
d) Drying shrinkage of concrete.

Shrinkage from early thermal effects occurs in the first four days of casting and although common to both reinforced and prestressed concrete it is of a similar order to elastic shortening from prestressing. Elastic shortening occurs during stressing of the tendons, but the creep and drying shrinkage are long-term effects.

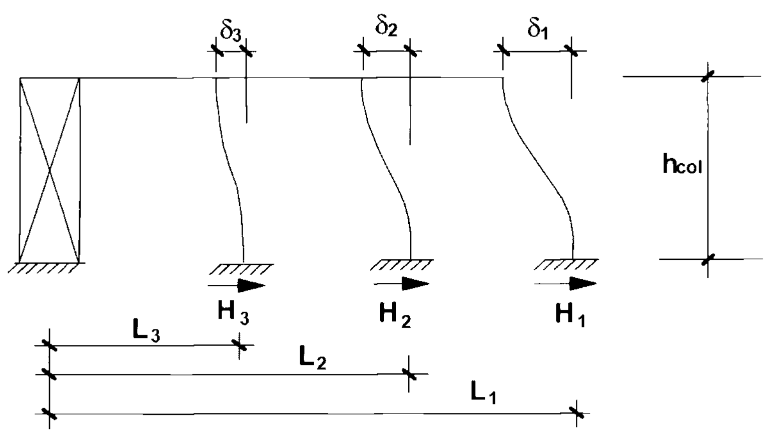
The floor is supported on columns or a combination of columns and core walls. These supports offer a restraint to the shortening of the floor. There are no firm rules that may be used to determine when such restraint is significant. As a guide, if the prestress is less than 2MPa, the floor is not very long (say less than 50m) and there is not more than one stiff restraint (e.g. a lift shaft), then the effects of restraint are usually ignored.

A simple method of ascertaining the restraint offered by the supports is to calculate the early thermal shrinkage, elastic, creep and drying shrinkage strains expected in the slab and then to calculate the forces required to deflect the supports. Figure 20 shows two simple frames in which the floors have shortened and the columns have been forced to deflect. The force in each column may be calculated from the amount it has been forced to deflect and its stiffness. The stiffness may be calculated on the assumption that the column is built-in at both ends.

The calculation of elastic, creep and shrinkage strains may be based on the values given in BS 8110[5]. The elastic strain should be based on the modulus of elasticity at the time the tendons are stressed. If this is at seven days after casting the modulus is approximately 80% of the modulus at 28 days. The creep strain depends on the age of the concrete when the tendons are stressed, the humidity and the effective thickness. The creep strain would be typically 2.5 times the elastic strain. The shrinkage strain will generally be in the range $100–300 \times 10^{-6}$, but in some circumstances it can increase to 400×10^{-6}.



a) Symmetrical floor supported on columns



b) floor supported by columns and lift shaft at one end

Figure 20: Restraint to floor shortening.

Typical strains for a 300mm internal floor with a prestress of 2MPa would be:

Early thermal shrinkage strain	100×10^{-6}
Elastic strain	100×10^{-6}
Creep strain	250×10^{-6}
Drying shrinkage strain	300×10^{-6}
Total long-term strain (ε_{LT})	750×10^{-6}

The following analysis is approximate but conservative and ignores any displacement of the foot of the columns or rotation of the ends of the columns. A more accurate analysis may be made using a plane frame with imposed member strains.

The force required to deflect each column, as shown in Figure 20, may be assumed to be calculated as follows:

$$\delta_i = \varepsilon_{LT} \times L_i$$

$$H_i = 12E_c\, I_i\, \delta_i \,/\, (h_{col})^3$$

For the purposes of calculating H_i, the value of $E_c\ I_i$ for the column may be reduced by creep in the column and in some cases cracking. A reduction of at least 50% from the short-term elastic properties is normally justifiable.

The total tension in the floor due to the restraint to shortening is the sum of all the column forces to one side of the stationary point. In Figure 20a, the tension is $H_1 + H_2$; in Figure 20b, the tension is $H_1 + H_2 + H_3$. This tension acts as a reduction in the pre-compression of the floor by the prestress. If the tension is small in comparison with the prestress, it may be ignored. If the tension force is significant, it may be necessary to subtract it from the prestress to obtain the effective pre-compression of the floor.

It should be noted that if the restraint is so severe that flexing of the vertical members to accommodate the shortening is not possible, other measures must be provided. These may include freeing the offending stiff elements during a temporary condition. However, it should also be remembered that creep and shrinkage will continue to occur for up to 30 years.

3.4 DURABILITY AND FIRE RESISTANCE

The durability and fire requirements may affect the choice of layout and form of the floor.

BS EN 1992-1-1[(7)], Table 4.1 provides exposure classes related to environmental conditions in accordance with BS EN 206-1[(12)] and BS 8500 [(13)]. Durability is controlled largely by the cover to reinforcement and prestressing tendons (see Chapter 6 of this Report).

BS EN 1992-1-2[(7)] provides information concerning the fire resistance of concrete floors. Fire resistance is controlled largely by the cover to reinforcement and prestressing tendons, and the thickness of floor (see Chapter 6 of this Report).

4 MATERIALS

4.1 CONCRETE

Concrete should be specified in accordance with BS EN 206-1[12] and the associated BS 8500[13] (previously Parts 1 and 2 of BS 5328[14]). It should be mixed and transported in accordance with Part 3 of BS 5328 and placed in accordance with the National Structural Concrete Specification[15]. The choice of concrete type and grade will be influenced by durability requirements, early strength gain requirements, material availability and basic economics. At present concrete grades of C30/37 and C35/45 are the most commonly used for post-tensioned floors. Strength at transfer of prestress is required at typically four to seven days. This normally means that the 28-day strength needs to be over C30/37.

Where lightweight aggregates are used, references should be made to the special requirements of Section 11 of BS EN 1992-1-1[7].

4.2 TENDONS

4.2.1 Strand

The tendon material used for post-tensioning concrete floors is normally 7-wire strand. Commonly used strand in the UK is shown in Table 2.

4.2.2 Tendon protection

Unbonded tendons

Unbonded tendons are protected by a layer of grease inside a plastic sheath. An example is shown in Figure 21.

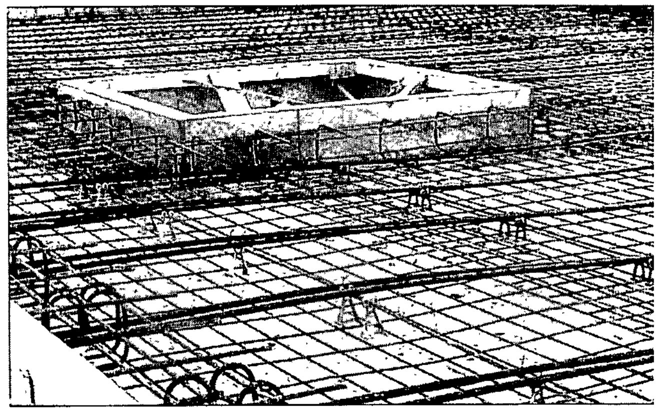


Figure 21: Layout of unbonded tendons.

These materials should comply with the recommendations given in the draft BS EN 10138[16].

Under normal conditions, the strand is supplied direct from the manufacturer already greased and sheathed. In no circumstances should PVC be used for the plastic sheath, as it is suspected that chloride ions can be released in certain conditions.

Bonded tendons

Bonded tendons are placed in metal or plastic ducts, which can be either circular or oval in form. An example is shown in Figure 22. The oval duct is used in conjunction with an anchorage, which ensures that between four and six strands are retained in the same plane in order to achieve maximum eccentricity.

Table 2: Specification of commonly used strand in the UK.

Strand type	Steel number	Nominal tensile strength (MPa)	Nominal diameter (mm)	Cross-sectional area (mm^2)	Nominal mass (kg/m)	Characteristic value of maximum force (kN)	Maximum value of maximum force (kN)	Characteristic value of 0.1% proof force (kN)
12.9 'Super'	Y1860S7 1.1373	1860	12.9	100	0,781	186	213	160
15.7 'Super'	Y1770S7 1.1375	1770	15.7	150	1.17	265	302	228
15.7 'Euro'	Y1860S7 1.1373	1860	15.7	150	1.17	279	319	240
15.2 'Drawn'	Y1820S7G 1.1371	1820	15.2	165	1.290	300	342	258
Note: The table is based on information from BS EN 10138-3[16]								

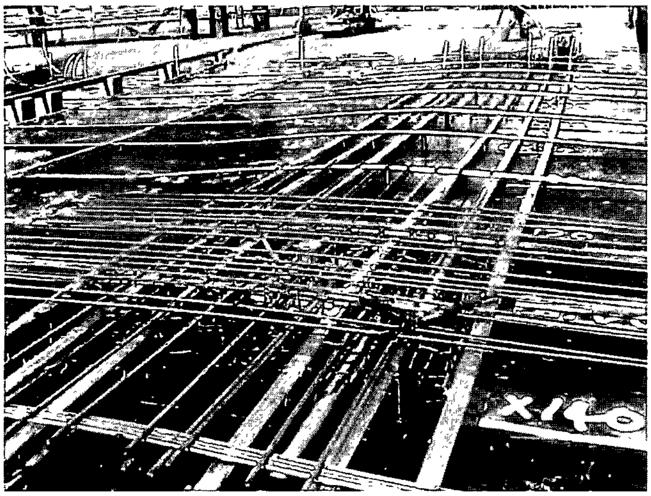


Figure 22: Layout of bonded tendons

Metal ducts are made from either spirally wound or seam-folded galvanised metal strip. On completion of stressing, the ducts are pumped full of cement grout which effectively bonds the strand to the structure as well as ensuring corrosion protection. This procedure should be carried out in accordance with the National Structural Concrete Specification (NSCS)[15]. Grouting should be in accordance with BS EN 445, 446 and 447[17-19].

While metal ducts are acceptable for internal environments, plastic ducts should be considered for external environments, especially where de-icing salts are present. When considering the use of plastic ducts the following should be taken into account:

- Exposure – Will a waterproofing layer be used, will this be maintained, what is the distance from the source of de-icing salts etc?
- Criticality – How sensitive is the structure to corrosion occurring within a duct? Bridges have relatively few ducts and so corrosion in one duct is likely to be more significant than in a slab with a number of ducts. Nonetheless loss of a duct's worth of tendons would be significant for a post-tensioned slab and, with steel ducts, inspection of ducts by non-intrusive methods is difficult.
- System requirements – How far do you adopt the bridge type approaches described in Concrete Society Technical Report 47[10]? This recommends that plastic ducts are used in addition to pressure testing of each duct and plastic caps to the anchorages. Pressure testing each duct within a post-tensioned slab would be very time consuming, however some testing to demonstrate that the system provided a barrier to chlorides would be appropriate.
- Overall durability – What is the most sensitive detail? Post-tensioned slabs normally have passive reinforcement in addition to the prestressing tendons. If the tendons are in a plastic duct then this passive reinforcement may become the critical element. While problems with reinforcement corrosion are more obvious and easier to repair it would be more appropriate to ensure the whole structure had a similar level of reliability.
- Economics – What cost premium is the client prepared to pay for the additional reliability? A post-tensioned slab with tendons in fully tested plastic ducts should provide a more durable slab than a normal reinforced concrete slab by minimising the unprotected reinforcement. Currently the cost of the post-tensioned slab with plastic ducts would be greater than that of a post-tensioned slab with traditional steel ducts. Post-tensioned slabs are often proposed as alternatives for reinforced concrete slabs and the use of plastic ducts will make them less attractive if considered on cost grounds alone.

4.2.3 Anchorages

Anchorage components should comply with BS 4447[20]. Details of these are shown in Figures 23 and 24. In the case of anchorages for unbonded tendons corrosion protection should comply with Class A exposure as defined in *Recommendations for the acceptance and application of post-tensioning systems*[21]. In addition, tests for unbonded anchorages should include fatigue testing consisting of cycling the prestressing force between 60% and 65% of the characteristic strength of the strand for two million cycles.

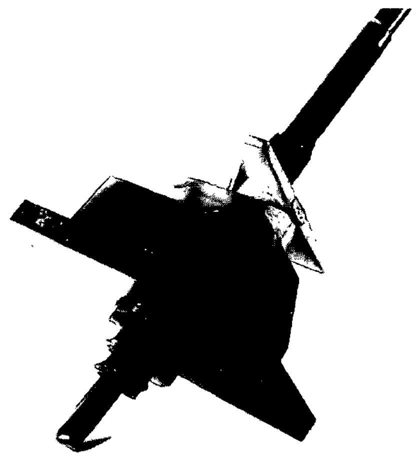


Figure 23: A typical anchorage for an unbonded tendon.

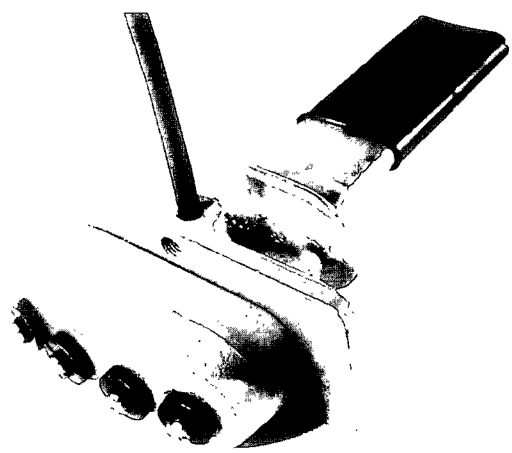


Figure 24: A typical anchorage for a bonded tendon.

4.3 UN-TENSIONED REINFORCEMENT

Un-tensioned reinforcement should comply with BS 4449[22] and the draft BS EN 10080[23].

5 THE DESIGN PROCESSS

5.1 INTRODUCTION

A typical design flow chart is shown in Figure 25 overleaf.

This chapter considers the various stages of the design process in more detail. As in most reinforced and prestressed concrete design work, the customary design process is of an iterative nature following the cycle:

1. Carry out preliminary design.
2. Check design with analysis.
3. Revise design as required.
4. Repeat steps 2 and 3 if necessary.

It should be clearly stated in writing for each contract who is responsible for the design, the specification, the detailed calculations and the working drawings for the prestressed elements. In addition it should be made clear who is responsible for co-ordinating the interfaces between the elements and how this relates to the overall responsibility for the design of the structure.

The analysis may be based on semi-empirical procedures such as the 'equivalent frame' method or more rigorous analysis such as grillage or finite element methods. The use of yield line analysis does not take account of the advantages of prestressing for the Serviceability Limit State.

The design is assumed to be in accordance with BS EN 1992 -1-1[(7)] (Eurocode 2) and is based on concrete cylinder strength, f_{ck}. Additional guidance is given in this Report. For flat slabs the depth of slab is often controlled by its shear capacity. Otherwise, in this design guide, the flexural design at Serviceability Limit State (SLS) is considered first, followed by checks on flexural and shear capacity at Ultimate Limit State (ULS).

5.2 STRUCTURAL LAYOUT

The choice of layout and member sizing has been discussed in Chapter 3, and is probably the most important decision in the design process. Unless previous experience or overriding factors dictate the exact form and section, several possibilities should be studied, although the designer should be able to limit the possible solutions by considering the various constraints and by rough design and costing exercises. With regard to slab thickness and concrete strengths, the relationship of structural layout, slab thickness and loading has been referred to in Chapter 3. Typical span/depth ratios are given in Table 1. A determination of a trial member depth should be made at an early stage in the calculation process. A general guide is to assume a depth of about 70% of the equivalent non-prestressed member.

5.3 LOADING

For Serviceability Limit State the dead load and post-tensioning effects, including the effect of losses due to creep, long-term shrinkage and relaxation of the prestressing steel, should be considered as acting with those combinations of live loads which result in the maximum stresses. Unless there are specific abnormal loads present, it will generally be sufficient to consider the post-tensioning effects in combination with the live loads as given in Eurocode 2, Clause 5.1.2 (see UK National Annex). For flat slabs it is normally satisfactory to apply the combinations of loading to alternate full width strips of the slab in each direction (not 'chequer-board'). However it will normally be satisfactory to obtain the moments and forces under the single load case using the frequent load values, provided that the limitations set out in the UK National Annex are satisfied.

Where the analysis is used to determine deflections, span/500 is normally an appropriate limit for quasi-permanent loads (see Eurocode 2, Clause 7.4.1). It may be necessary to consider other limits and loads depending on the requirements for the slab (see also Section 5.8.4).

Where the analysis is used to determine crack widths the frequent load combination should be used (for bonded or unbonded tendons). This is in accordance with the UK National Annex to Eurocode 2 and is checked against a maximum permitted crack width of 0.3mm. This limit is given to ensure an acceptable appearance. Other crack width limits may be specified by the client.

The use of the characteristic combination should be subject to client's requirements and engineering judgement. It should only be used when there are parts of the building that would suffer from an irreversible change (e.g. brittle floor finishes, brittle partitions, brittle facades etc).

At transfer of prestress the dead loads present during stressing, together with the post-tensioning effects and the effects of early thermal shrinkage, should be considered in obtaining stresses.

Where the applied loads change significantly during construction or phased stressing is employed, the various stages should each be checked for transfer stress limits.

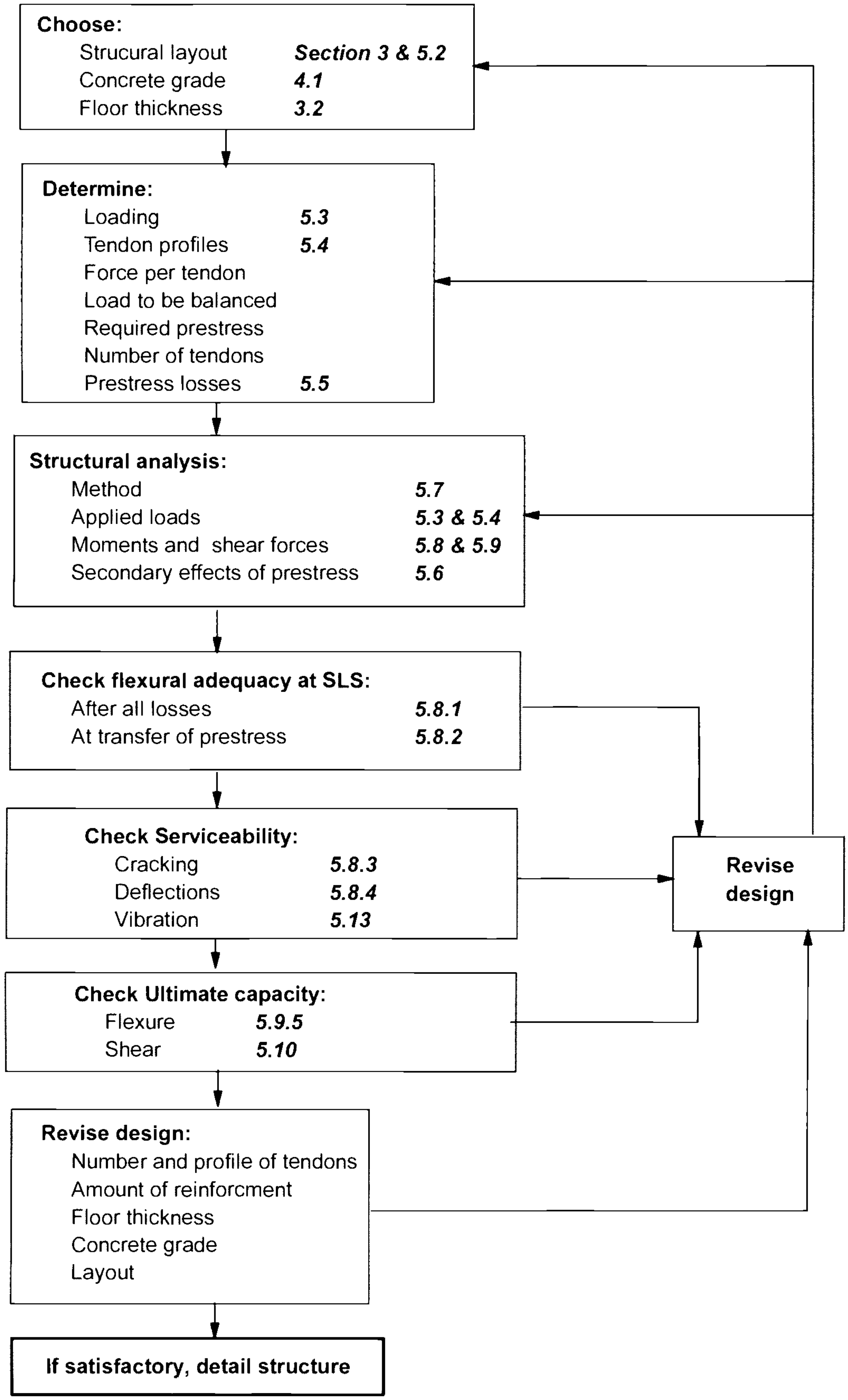


Figure 25: Design flow chart.

At the ULS the load combinations shown in Eurocode 2, Clause 5.1.2 should be used to arrive at the maximum moments and shears at any section. When checking flexural stresses, secondary effects of prestressing may be included in the applied loads with a load factor of 1.0 (see Section 5.8). However for the shear resistance check of members other values should be used (see Section 5.9).

5.4 TENDON PROFILE AND EQUIVALENT LOAD

Ideally the tendon profile is one that will produce a bending moment diagram of similar shape, but opposite sign, to the moments from the applied loads. This is not always possible because of varying loading conditions and geometric limitations.

It should be noted that for bonded systems the centroid of the strands will not coincide with the centroid of the duct. This is particularly true in the case of circular ducts. Further information may be available from the manufacturer's literature.

In the simplest case, for a uniformly loaded simply-supported beam, the bending moment is parabolic, as is the ideal tendon profile as shown in Figure 26.

The total 'sag' in the parabola is referred to as the tendon 'drape', and is limited by the section depth and minimum cover to the tendon. At the supports the tendon has no eccentricity and hence there is no bending moment due to the tendon forces.

Tendon profiles are not always symmetric. However, the point of maximum drape is still at the centre of the points of inflection, but may not correspond to the point of maximum sag (see Figure 27).

The upward forces applied to the concrete by a parabolic profiled tendon, as shown in Figure 26, are uniformly distributed along the tendon. At the ends of the tendon downward forces are applied to the concrete by the anchorages. The upward and downward forces are in equilibrium so that no external forces occur. The set of forces applied to the member by the tendon are known as the 'equivalent' or 'balanced' loads, in that the upward forces counterbalance a proportion of the downward forces due to dead and live loads.

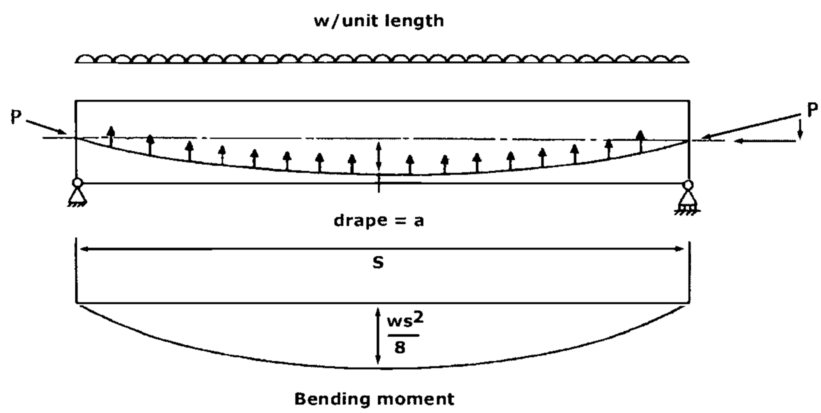


Figure 26: Idealised tendon profile.

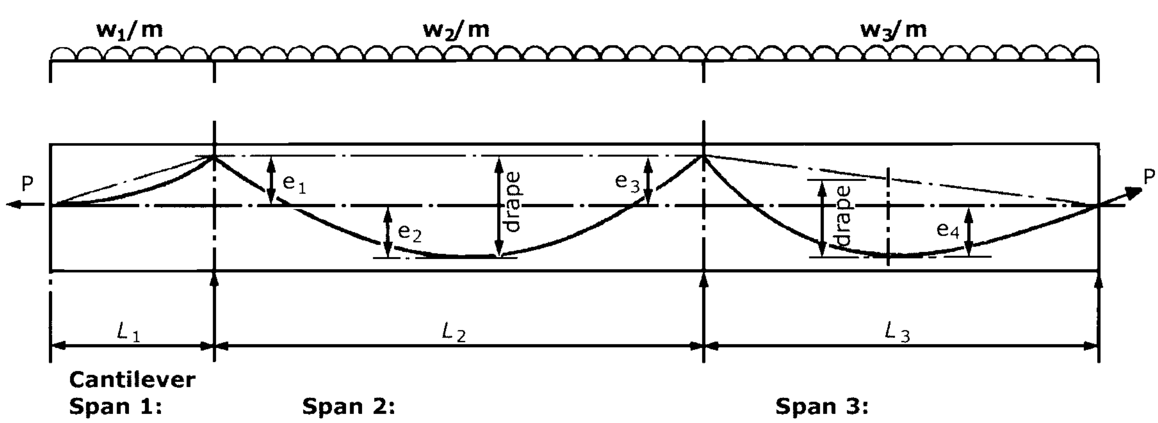


Figure 27: Idealised tendon profile for two spans with single cantilever.

For a parabolic profile the upward uniformly distributed load, w, can be calculated as follows:

$$ws^2/8 = P_{av}\,a$$

or

$$w = 8aP_{av} / s^2$$

where
- s = distance between points of inflection
- a = drape of tendon measured at centre of profile between points of inflection. Note that this may not be position of maximum sag
- P_{av} = average prestressing force in tendon.

Usually, in continuous members, the most effective use of a tendon in producing 'balanced loads' is achieved by having the tendon at its lowest possible point in positive moment locations, and at its highest possible point in negative moment locations (see Figure 27). In this way the drape, and consequently the 'balanced loads', is increased to a maximum.

The 'equivalent' or 'balanced' loads may be applied to the structural frame in order to obtain the effects of prestressing. Some typical 'equivalent' loads are given in Figure 28.

The effects of equivalent loads include primary and secondary effects as described in Section 5.6.

In post-tensioned design it is common to roughly 'balance' equal proportions of the dead and applied loads in each span. Some designers set out with a preconceived idea of what load they wish to balance as a proportion of the dead or total load. Others balance the minimum amount which will result in the final stresses due to the out-of-balance loads being as close as possible to the maximum allowable stresses.

This latter approach is usually the most economical overall but may not always be the most suitable for deflection or congestion of un-tensioned reinforcement.

Figure 27 illustrates an idealised tendon profile for a two-span member with a cantilever. The parabolic profiles result in the balanced loads w_1, w_2 and w_3 as shown, calculated from the tendon profile and hence the 'drapes'.

Figure 29 illustrates a two-span member with an idealised tendon profile to provide a uniform uplift over span 1 and a concentrated uplift in span 2. The concentrated effect is useful in members transferring column or similar point loads.

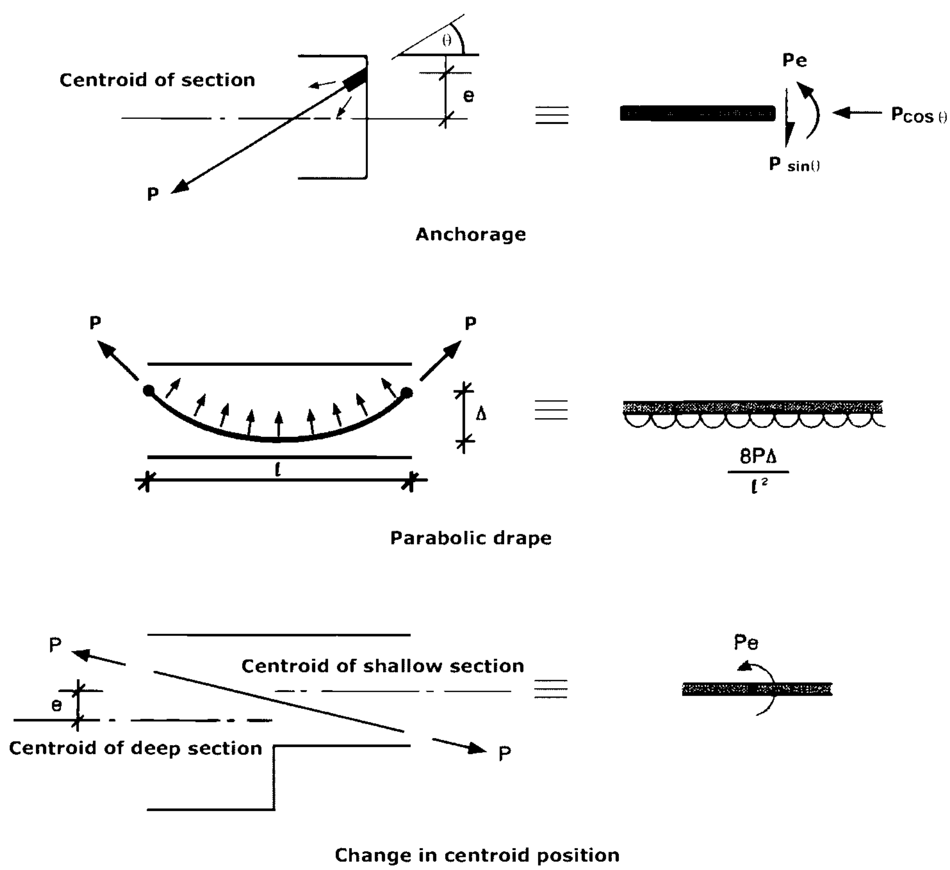


Figure 28: Typical prestressing tendon equivalent loads.

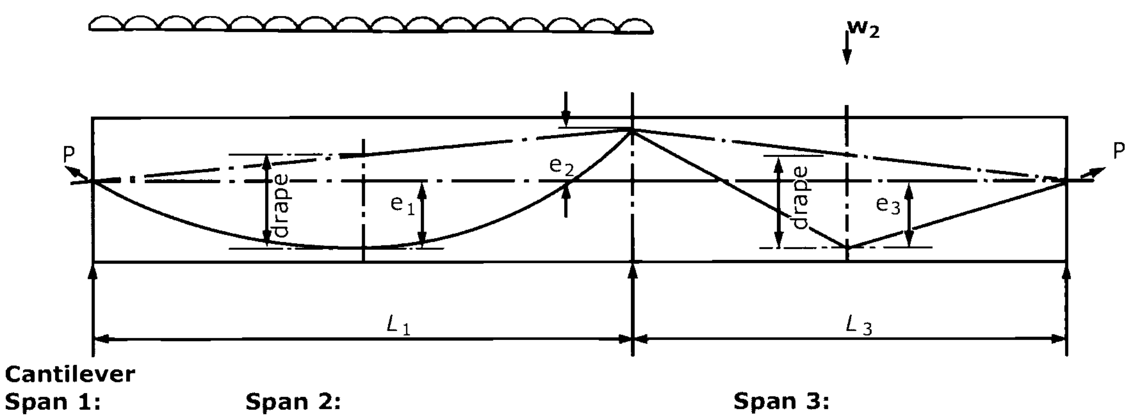


Figure 29: Idealised tendon profile for two spans with point load.

Note to Figure 29: The centroid of the concrete and the centroid of the tendon coincide at the end of the member to ensure that no equivalent moments are applied at the end.

Span 1: Span 2:

Total drape = $e_1 + e_2 / 2$	Total drape = $e_3 + e_2 / 2$
Equivalent UDL = $P \times$ total drape $\times 8/L_1$	Equivalent point load = $P \times$ total drape $\times 4/L_2$

where
P = prestressing force at the section under consideration.

While the bending moments 'peak' over the supports, it is clear that in practice a tendon cannot do this and some approximation must be made. Remember that the peak is where the tendon is 'dumping' the load it has picked up by its parabolic shape (see Figure 30). In practice, tendon profiles are of the form shown in Figure 31.

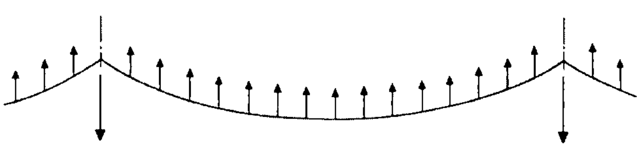


Figure 30: Local 'dumping' at 'peaks'.

The ratio L'/L should generally be kept as small as possible (e.g. 0.05 for $L/d = 40$). Unless the specialist literature states otherwise for multi-strand circular ducts the radius should not be less than 70 × the duct diameter and for flat ducts the radius should not be less than 2.5m.

Appendix C provides information from which the parabolic tendon geometry can be calculated.

The resultant balancing forces are therefore as shown in Figure 32.

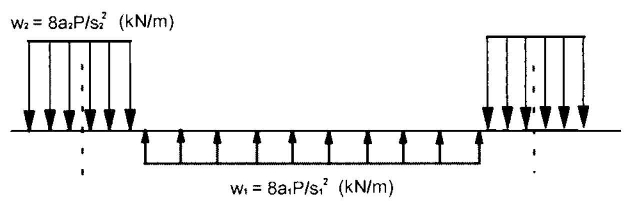


Figure 32: Resultant balancing forces.

For the reverse parabola at the support the total force downwards:

$$W_2 \quad = \quad w_2\, s_2 = 8Pa_2 / s_2$$

and for the span parabola the total load upwards:

$$W_1 \quad = \quad w_1\, s_1 = 8Pa_1 / s_1$$

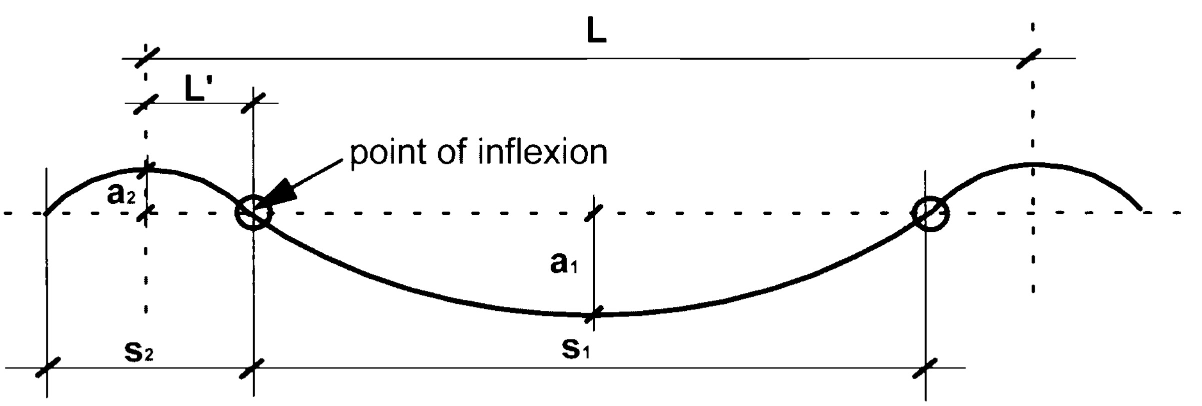


Figure 31: Practical representation of idealised tendon profile.

If L'/L is made equal to 0.1, as suggested above,

then $s_1 = 4s_2$

Since the upward and downward loads must be equal, it follows that:

$a_1 / s_1 = a_2 / s_2$

and hence:

$a_1 = 4a_2$

The equivalent loads upwards and downwards due to the tendons can thus be calculated.

5.5 PRESTRESS FORCES AND LOSSES

From the time that a post-tensioning tendon is stressed, to its final state many years after stressing, various losses take place which reduce the tension in the tendon. These losses are grouped into two categories, namely short-term and long-term losses.

5.5.1 Short-term losses

The short-term losses include:

a) Friction losses in the tendon
b) Wedge set or 'draw-in'
c) Elastic shortening of the structure.

These losses take place during stressing and anchoring of the tendon.

5.5.2 Long-term losses

The long-term losses include:

a) Shrinkage of the concrete
b) Creep of the concrete including the effect of the prestress
c) Relaxation of the steel tendon.

Although these losses occur over a period of up to ten or more years, the bulk occurs in the first two years following stressing. The loss in prestress force following stressing can be significant (between 10% and 50% of the initial jacking force at transfer and between 20% and 60% after all losses) and therefore the losses should, in all instances, be calculated in detail using the methods given in Appendix B.

5.6 SECONDARY EFFECTS

The secondary effects of prestressing are sometimes called 'parasitic effects' but that implies that the effects are unwanted and harmful. This is not in fact the case. For most structures the secondary moment will be a sagging moment and will increase the moments due to applied loads at mid-span but reduce the moments at the support. In some structures it is possible to 'tune' the secondary effects by adjusting the shape of the tendon profile to obtain the optimum solution. This is more likely to be of use in the design of beams rather than slabs.

Primary prestressing forces and moments are the direct result of the prestress force acting at an eccentricity from the section centroid. The primary moment at a section is simply the sum of the products of each tendon force with its eccentricity; the primary shear is the sum of transverse components of the tendon forces and the primary axial load is the sum of the axial components of the tendon forces.

When an element of a structure is prestressed, this causes its shape to change. It will always shorten, and will bend if the centroid of the prestress force does not coincide at all positions with the section centroid. (It is possible, however, to select a tendon profile which results in no rotation of the element ends.)

If the element is part of a statically determinate structure then these changes in shape will not affect the distribution of forces and moments (see Figure 33).

But when the element forms part of an indeterminate structure, the changes in shape resulting from prestressing will modify the support reactions. Additional reactions are required to make the prestressed member pass through support points and have suitable orientation where appropriate (see Figure 34).

These secondary reactions result in secondary forces and moments in the members. These are typically constant axial and shear forces throughout a span and uniformly varying moments. The calculation of these secondary effects can be difficult, when staged construction, creep and shrinkage are considered. (Note that secondary effects cannot develop in cantilevers as they are statically determinate.) Methods of calculating secondary effects are given in Appendix D.

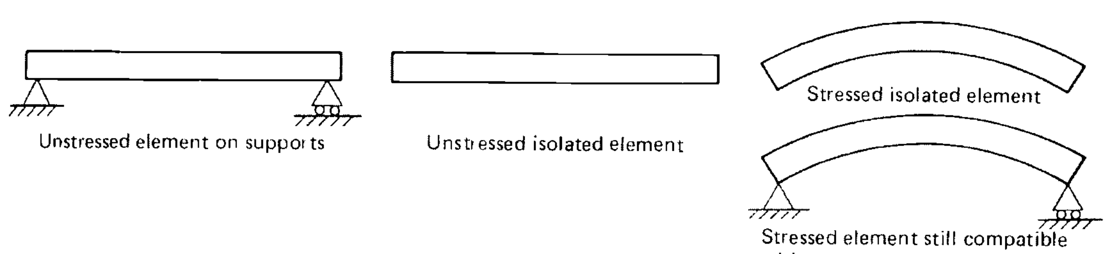


Figure 33: Prestressed element as a part of a statically determinate structure.

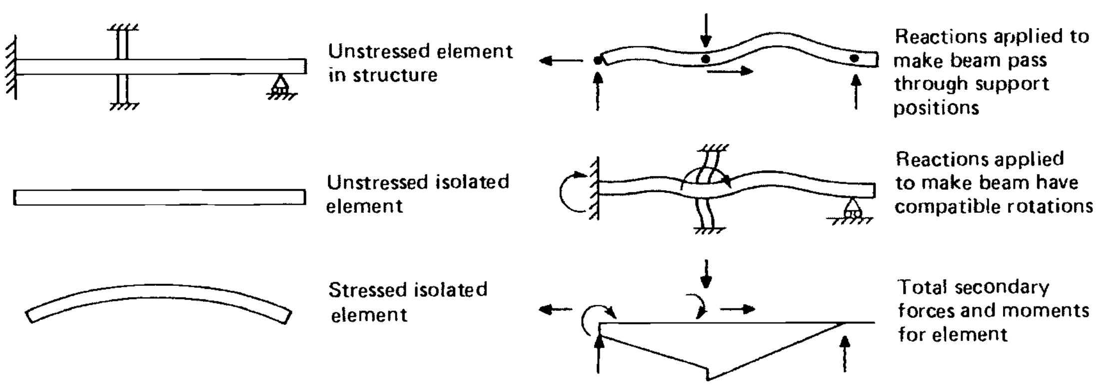


Figure 34: Reactions on a prestressed element due to secondary effects.

Equivalent loads will automatically generate the primary and secondary effects when applied to the structure.

Serviceability calculations do not require any separation of the primary and secondary effects, and analysis using the equivalent loads is straightforward. However, at ULS the two effects must be separated because the secondary effects are treated as applied loads. The primary prestressing effects are taken into account by including the tendon force in the calculation of the ultimate section capacity. The primary prestressing forces and moments must therefore be subtracted from the equivalent load analysis to give the secondary effects.

To calculate the ultimate loading on an element, the secondary forces and moments are combined with the ultimate forces and moments from dead and live loads. It will normally be satisfactory to use a partial load factor of 1.0 for secondary effects when calculating the flexural stresses where linear analysis with uncracked sections is applied. However for calculating the shear resistance other partial factors should be used (see Section 5.9).

5.7 ANALYSIS OF FLAT SLABS

5.7.1 General

The analysis of post-tensioned flat slabs differs from a reinforced concrete design approach owing to the positive effect that the tendons have on the structure. In reinforced concrete the reinforcement is initially unstressed; the stress in the reinforcement results from the deformation and cracking of the structure under applied load. In this way the reinforcement may be considered to act passively. On the other hand, the tendons in a post-tensioned floor are actively stressed by the jacks so that they are loaded before the application of other loads with the exception of early thermal shrinkage. The force in the tendon is chosen by the designer (e.g. to balance the unfactored dead load). At ULS the force in unbonded tendons does not increase significantly from that of the initial prestressing force, in contrast to the force in bonded tendons, which reaches the yield strength at critical design sections.

The 'equivalent frame' method of analysis may be undertaken by hand, using moment distribution or flexibility methods. It is common to analyse structures using plane frame computer programs. However, when longhand moment–distribution calculations are employed, stiffness, carry-over factors and fixed end moment coefficients must be calculated. These can be quite complicated for varying sections, column heads and drop-panels and, although often ignored in hand calculations, the effect on stiffness of the complete beam second moment of area over the column width can be most significant, particularly for wide columns.

There are also available on the market several computer programs and spreadsheets specially written for post-tensioned flooring systems. These programs not only undertake the analysis of the frame under applied loading and loading from the tendons, but also calculate the flexural stresses.

Grillage and finite element programs are now available which are more suitable for complex flat slabs and slabs with irregular column layouts.

Whichever technique is used for the structural analysis it must take into account not only the dead and live loads but also the loads that the tendons apply to the structure (see Section 5.6).

It is considered reasonable that, for flat slabs, hogging moments greater than those at a distance $h_c/2$ from the centreline of the column may be ignored provided that the sum of the maximum positive design moment and the average of the negative design moments in any span of the slab for the whole panel width is not less than:

$$nl_2\,(l_1 - 2h_c/3)^2 / 8$$

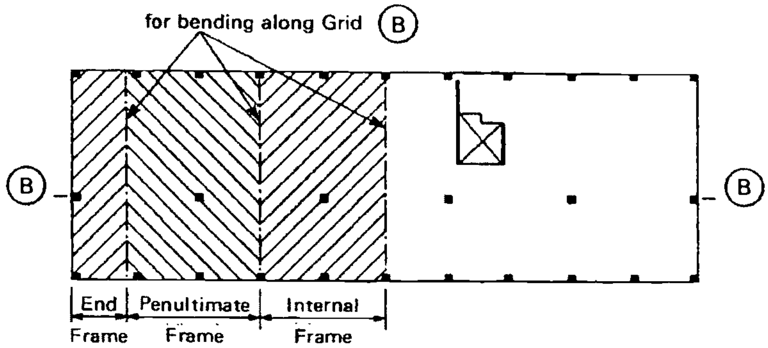
where

n = design ultimate load on the full width of panel between adjacent bay centre-lines

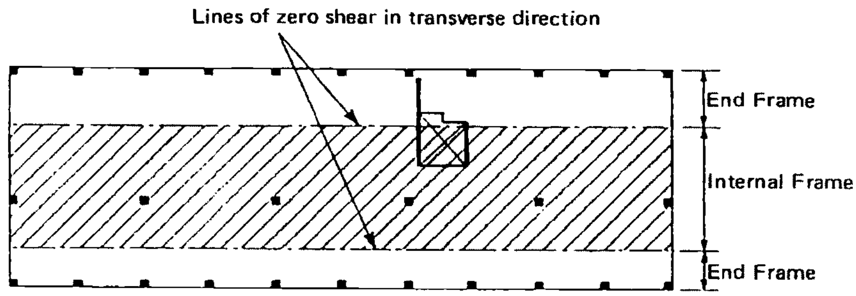
l_1 = panel length parallel to span, measured from centres of columns

l_2 = panel width, measured from centres of columns

h_c = effective diameter of a column or column head.



a) Equivalent frame widths for frames spanning in the transverse direction



b) Equivalent frame widths for frames spanning in the longitudinal direction

Figure 35: Elastic load distribution effects.

5.7.2 Equivalent frame analysis

It is common to divide the structure into sub-frame elements in each direction. Each frame usually comprises one line of columns together with beam/slab elements of one bay width. The frames chosen for analysis should cover all the element types of the complete structure.

The ends of the columns remote from the sub-frame may generally be assumed to be fixed unless the assumption of a pinned end is clearly more reasonable (e.g. pad footings). Equivalent frame analysis for flat slabs does not take account the extra flexibility at the junction of the slab and edge columns. In order to simulate this it may be appropriate to use an equivalent length, kl_{act}, of column larger than the actual length, l_{act}, where k = 0.5(Column spacing) / (Column width + 6 × depth of slab).

The use of the equivalent frame method does not take account of the two-dimensional elastic load distribution effects automatically. It will give different support reactions from the analyses in the two orthogonal directions unless the width of slab chosen coincides with the points of zero shear in the other direction. Normally for internal bays the width of slab will be the full panel width. However for a regular layout, the penultimate frame will pick up more than half the width on the side of the end bay (see Figure 35).

Provided the reaction on each column is taken as the larger value from the two analyses, little accuracy will be lost. However where the size and arrangement of edge columns is different from the internal columns, the width of slab should be estimated more accurately. This will ensure the correct selection of the number of prestress tendons with the profile appropriate for the frame being analysed.

It should be noted that these elastic effects are automatically taken into account when the floor is analysed using grillage or finite element methods.

Irrespective of which analytical technique is used, care should be taken to ensure that the assumptions made are appropriate to the structure under consideration. In particular the prestress applied to two adjacent frames should not be very dissimilar otherwise the prestress from the more highly stressed frame will dissipate into the adjacent frames.

Eurocode 2, Annex I, describes how the applied bending moments (excluding prestressing effects) are distributed between 'column' and 'middle' strips within a flat slab with a simple orthogonal layout of columns. It also suggests a simple method of applying load combinations to a slab with irregularly placed columns. Other methods may also be used provided that they simulate the actual behaviour reasonably well.

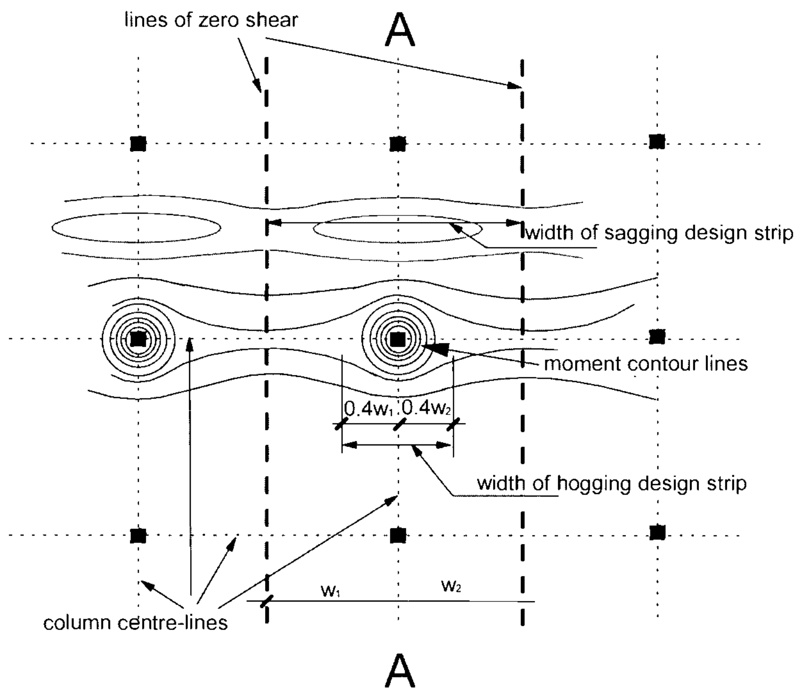


Figure 36: Typical distribution of bending moments about the x-axis along column line A–A for uniformly distributed loading and a regular column layout.

Note to Figure 36: The nominal widths of the hogging design strips should be calculated as shown above for the two design directions at a column and the lesser value should be used for the design in both directions at that column.

5.7.3 Finite element or grillage analysis

The use of finite element or grillage programs for analysis of flat slabs is normally based on the elastic properties of the concrete section and the guidance given here assumes an elastic distribution of moments and stresses.

The design of flexural reinforcement may be based on moment contours about two orthogonal directions. Typical moment contours for moments about the x-axis along the column line A–A for uniformly distributed loading (excluding prestressing effects) are illustrated in Figure 36. 'Design strips' can be set up for the critical sagging and hogging areas of the slab to determine the required reinforcement. The following rules apply for regular layouts of columns. For irregular layouts of columns similar rules, using engineering judgement, may be followed. It should be noted that where moments with opposite sign occur within a single strip these should not generally be averaged.

First, lines of 'zero shear' for flexure in the 'x-direction' (i.e. about the y-axis) are located. The 'design strips' are based on these and the column centre lines. The 'zero shear' lines should be determined using the ULS load combination.

Sagging areas

The moments across a sagging area do not vary sharply and for the purposes of design the moments and reinforcement (if required) may normally be considered to be distributed evenly across the full width. The width of the design strip for sagging moments may be taken as the distance between lines of zero shear (see Figure 37, 'design strip' No. 1). Where the reinforcement and bonded tendons are not evenly spaced across this width, the sagging design strip should be divided into separate strips for crack control design. This applies both for checks based on gross section properties using Tables 3–5 in Section 5.8.1, which are dependent on the presence of bonded reinforcement near the tension face, and for cracked section checks, which are dependent on the area of bonded reinforcement in the vicinity of the crack.

Where the slab is designed without bonded tendons or reinforcement, the stress limits given in Table 5 should be used. Where the slab is designed using bonded tendons and/or reinforcement, the limit given in Table 5 for 'with bonded reinforcement' may be used provided that the spacing of the tendons or bars does not exceed 500mm. Otherwise the stress limit for 'without bonded reinforcement' should be used. Where the designer chooses to calculate the crack width, this should be in accordance with Eurocode 2, Clause 7.3.4.

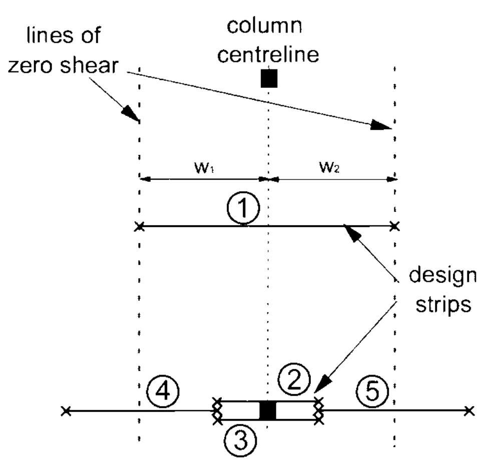


Figure 37: 'Design strips' for moments about the x-axis of typical flat slabs.

Hogging areas

The moments across hogging regions are sharply peaked over the column. There are typically four 'design strips' in the hogging areas at each column as shown in Figure 37 (Nos 2–5). The two primary hogging 'design strips' occur at each of the opposite faces of the column (Figure 37, line Nos 2 and 3) and the width of these may be taken as $0.4(w_1 + w_2)$ where w_1 and w_2 are the distances between the column centre line and the line of zero shear on each side of the column as shown in Figure 38, but not greater than the sum of the widths of the primary hogging design strips in the transverse direction at that column. For the purposes of design the moments and reinforcement may be taken as the mean across this width. Where the values of w_1 and w_2 differ, the mean values of moment may be different on each side of the column. The remaining 'hogging' region may, in fact, not be hogging across the full width of this part of the slab and the two secondary 'design strips' (Figure 37, line Nos 4 and 5) are placed in line with the centre line of the column with a width which extends either side of the line of 'zero shear'. Care should be taken to ensure that the maximum sagging and hogging moments (rather than the average values) can be adequately resisted by the slab in this area.

These assumptions are generally applicable for slabs designed to SLS and checked against the permissible stresses given in Section 5.8. Where the stresses are exceeded a calculation may show that the crack width is within the required limit. Otherwise changes in the design are required (e.g. the addition of reinforcement to limit crack widths). At ULS it is accepted that cracking will occur together with redistribution of the peaked moments.

5.7.4 Analysis for the load case at transfer of prestress

For post-tensioned slabs it is also necessary to carry out stress checks for the load case at transfer of prestress. It is generally acceptable to use similar design sections to those established for the SLS. However it should be noted that additional design sections may be required at locations to correspond with the particular stresses induced by the transfer loads.

5.7.5 Analysis for non-uniform loads

The above recommendations can also be applied to non-uniform loads. However the designer should be aware that additional design sections may be required to cater for peak bending stresses which occur at unusual locations, for example beneath any heavy point load. Additional design strips may also be required at the face of any changes in slab cross-section, for example if column head drops or beams are present. Design strips should also be added adjacent to any significant openings in the slab. The position and length of the design strip should be selected using engineering judgement based on the guidelines outlined above.

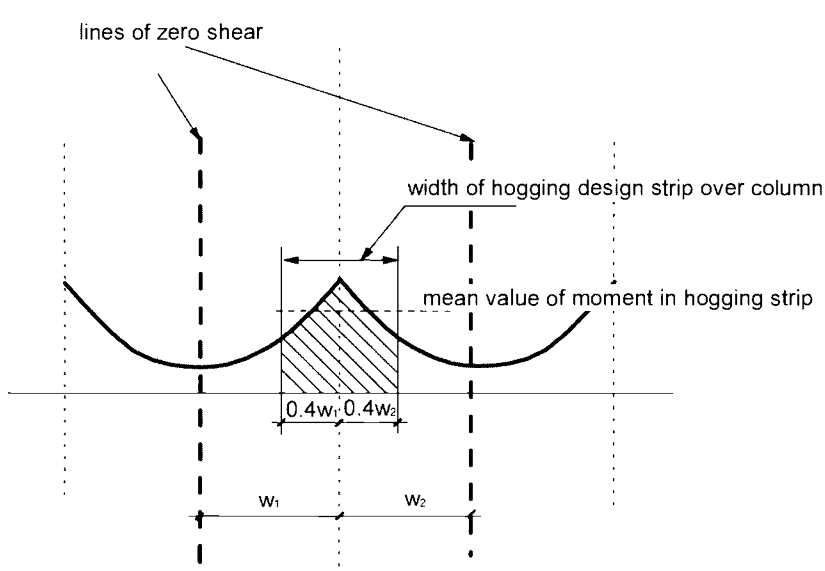


Figure 38: Section through moment diagram at column position.

5.8 FLEXURAL SECTION DESIGN

5.8.1 Serviceability Limit State: stresses after all losses

The bending moments calculated from the critical loading conditions given in Section 5.4, including the tendon effects, provide the serviceability stresses at each section using:

top fibre stress: $f_t = P / A_c + M / z_t$

bottom fibre stress: $f_b = P / A_c - M / z_b$

where

z_t = top section modulus

z_b = bottom section modulus

M = total out-of-balance moment

= $M_A - Pe + M_s$

e = eccentricity of tendons, taken as positive below the neutral axis

M_A = applied moment due to dead and live loads

M_s = moment from prestress secondary effects.

Beams, one and two-way (not flat slabs) spanning floors

The maximum allowable concrete compressive stresses for floors with bonded tendons are given in Eurocode 2, Clause 7.2. Most buildings will perform satisfactorily provided that the crack width is limited to 0.2mm but the nature of the loading should be considered when deciding this (e.g. frequency and duration).

Bonded tendons

Although cracking is permitted for Exposure Classes X0, XC1, XC2, XC3 and XC4 it may be assumed that the design hypothetical tensile stresses exist at the limiting crack widths given in Eurocode 2, Table 7.1N. Table 3 gives limits to the design hypothetical tensile stresses under the frequent load combination. These limits are appropriate where steel relaxation losses, shrinkage (including early thermal shrinkage) and creep effects are not taken into account in the frame analysis. Where these effects are taken into account f_{ctm} may be replaced by $f_{ctm,fl}$.

Table 3: Design hypothetical tensile stress limits for cracked sections.

Group	Limiting crack width (mm)	Design stress
Bonded tendons	0.1	$1.35f_{ctm}$
	0.2	$1.65f_{ctm}$
Unbonded tendons	–	$1.35f_{ctm}$

Where additional reinforcement is contained within the tension zone, and is positioned close to the tension faces of the concrete, these design hypothetical tensile stresses may be increased by an amount that is in proportion to the cross-sectional area of the additional reinforcement (expressed as a percentage of the cross-sectional area of the concrete in the tension zone). For 1% of additional reinforcement, the stresses may be increased by 4MPa. For other percentages of additional reinforcement, the stresses may be increased in proportion up to a limit of $0.3f_{ck}$.

Unbonded tendons

The maximum design hypothetical tensile stresses should be limited to those given in Table 3. If the stresses are enhanced by increasing the un-tensioned reinforcement, crack widths and deflections should be rigorously checked in accordance with Eurocode 2, Clauses 7.3 and 7.4 and all concrete tension should be carried by un-tensioned reinforcement (see Section 5.8.7).

In the final design a cracked section analysis should be undertaken to ensure crack width limits are not exceeded. Crack width analysis may be carried out using the procedure for flat slabs (see below).

Flat slabs (two-way spanning on discrete column supports)

The distribution of moments for a flat plate (see Figure 10) shows that hogging moments across a panel are sharply peaked in the immediate vicinity of the column and that the moment at the column face is several times the moment midway between columns.

Flat slabs may be analysed using 'equivalent frames' in each direction or by grillage/finite element methods. It should be noted that the permissible stresses given in Table 4 apply where the stresses have been averaged over the full panel (e.g. using an 'equivalent frame' method). They are lower than those for one-way floors to allow for this non-uniform distribution of moments across the panel. The permissible stresses given in Table 5 assume a grillage or finite element (FE) analysis.

Additional designed un-tensioned reinforcement is required in the support zone of all flat slabs, and in the span zone of slabs using unbonded tendons where the tensile stress exceeds $0.3f_{ctm}$. The design of this reinforcement should be in accordance with Section 5.8.7.

Where the stresses have been averaged over the whole panel (e.g. using the 'equivalent frame' method) account should be taken of the peaking of the moments at the columns, described in Section 2.4. The analysis results in moments and stresses averaged across the width of the panel. These stresses should be limited to those given in Table 4. In this table the support zone is considered to be any part of the span under consideration within $0.2 \times L$ of the support, where L is the effective span. Outside of this zone is considered to be the span zone. The tension limits are appropriate where steel

Table 4: Allowable average stresses in flat slabs for full panel width.

Location	In Compression	In Tension	
		With bonded reinforcement[2]	Without bonded reinforcement
Support	$0.3f_{ck}$[1]	$0.9f_{ctm}$	$0.3f_{ctm}$
Span	$0.4f_{ck}$[1]		

Note: Bonded reinforcement may be either bonded tendons or un-tensioned reinforcement.
[1] If ductility check is carried out this limit may be exceeded
[2] The spacing of bars or tendons should be ≤ 500mm, otherwise the stress for 'without bonded reinforcement' should be used.

Table 5: Allowable stresses in flat slabs using 'design strip' approach.

Location	In Compression	In Tension	
		With bonded reinforcement[2]	Without bonded reinforcement
Support Span	$0.4f_{ck}$[1]	$1.2f_{ctm}$	$0.4f_{ctm}$

[1] If ductility check is carried out this limit may be exceeded
[2] The spacing of bars or tendons should be ≤ 500mm, otherwise the stress for 'without bonded reinforcement' should be used.

relaxation losses, shrinkage (including early thermal shrinkage) and creep effects are not taken into account in the frame analysis. Where these effects are taken into account f_{ctm} may be replaced by $f_{ctm,fl}$.

Where grillage or finite element methods are used particular care should be taken in modelling the column/floor intersection and in the interpolation of the results obtained. For each 'design strip' the stresses should be determined based on the concrete section properties. The tensile and compressive stresses should be compared with the allowable average stresses given in Table 5. The tension limits are appropriate where steel relaxation losses, shrinkage (including early thermal shrinkage) and creep effects have been specifically calculated. Where these affects are taken into account f_{ctm} may be replaced by $f_{ctm,fl}$.

Where the stresses in a 'design strip' do not exceed those in Table 5 the design may be considered adequate. Where the tensile stresses are exceeded the designer should undertake further checks to determine design crack widths as detailed in Section 5.8.3.

5.8.2 Serviceability Limit State: stresses at transfer

Transfer stresses should be checked for all floors. These are likely to be more onerous for floors with high imposed loads.

Unless un-tensioned reinforcement has been added, the stress limits will normally be the 'without bonded reinforcement' values, as any bonded tendons will normally be at the compression face at transfer.

Un-tensioned reinforcement should be calculated in a similar manner to the reinforcement for the Serviceability Limit State (see Section 5.8.7).

Beams, one and two-way (not flat slabs) spanning floors

The concrete compressive stresses at transfer should not exceed $0.5f_{ci}$ at the extreme fibre (or $0.4f_{ci}$ for near uniform stress distribution) where f_{ci} is the concrete cube strength at transfer. The tensile stresses should not exceed $0.72f_{ctm}$.

Flat slabs (two-way spanning on discrete column supports)

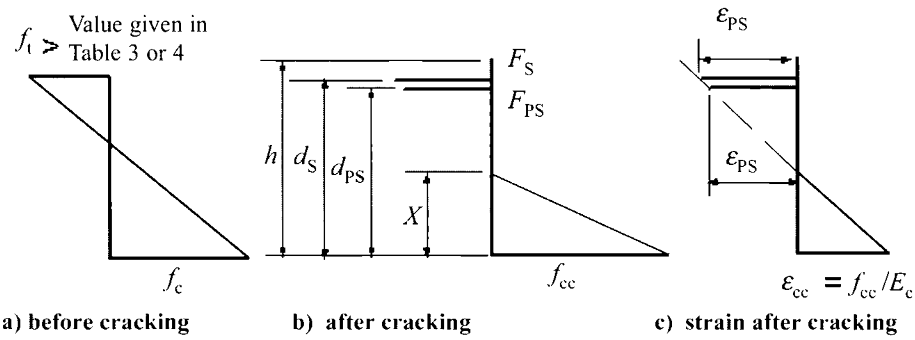
The design strips defined in Section 5.7.3 are considered appropriate for the checks at transfer. However, as noted, it may be necessary to add design strips specifically for the transfer condition.

The allowable stresses given in Table 5 are appropriate for the transfer condition. However f_{ck} should be replaced with f_{ci}. The stresses should be based on the concrete section.

Generally it is not recommended that the tensile stresses be allowed to exceed the maximum value in Table 5. However in extreme cases, for example heavily loaded transfer beams etc, a cracked section analysis, as detailed in Section 5.8.3 may be undertaken.

5.8.3 Crack width control

At locations where the tensile stresses in the design strip exceed the limits in Table 5 the designer has the option to either vary the design (slab thickness, prestress levels etc) or undertake an assessment of the crack widths. If, following this assessment, the crack widths are within the allowable limits given in Eurocode 2 then the design will be deemed satisfactory. The following method assumes no tensile stiffening in the concrete.



Note: The figure is drawn at a column location where the bending moments are negative and tension is in the top.

Figure 39: Assumed stress and strain distribution before and after cracking.

The stress block before cracking is shown in Figure 39a.

It is noted that for this situation the applied tensile stress, f_t, exceeds the allowable value from Table 5, namely $1.2f_{ctm}$. An equivalent cracked section is therefore established to maintain equilibrium with the forces and bending moments acting on the design strip (see Figure 39b).

The notation for Figure 39 is as follows:

f_{cc} = compressive stress in the cracked section ($\leq 0.4f_{ck}$ for linear stress distribution)
d_S = effective depth of the ordinary bonded reinforcement
d_{PS} = effective depth of the prestressing tendons
h = depth of the section
F_S = force in the ordinary bonded reinforcement
F_{PS} = force in the prestressing tendons
ε_{cc} = strain in concrete at extreme fibre $= f_{cc} / E_c$
E_c = Young's Modulus of the concrete (long-term value)
ε_S = strain in ordinary bonded reinforcement $= \varepsilon_{cc} \times (d_S - x) / x$
ε_{PS} = strain in prestressing strands $= \varepsilon_{cc} \times (d_{PS} - x) / x$

As the strains in the ordinary bonded reinforcement and the prestressing tendon can be derived from the strain in the concrete and the depth of the compression zone the expressions for the axial and bending equilibrium contain two unknowns: x and f_{cc}. These can be solved either numerically or by iterative processes and once known the stresses in the ordinary reinforcement and in any bonded prestressing tendons can be calculated. These values are then used to calculate the predicted crack width using the procedure detailed in Eurocode 2, Clause 7.3.4.

The permitted crack width should be taken as 0.2mm unless a more (or less) onerous value is specified by the client. Where the predicted crack width is less than the permitted value the serviceability stress/cracking checks will be deemed to be satisfied.

If the predicted crack width exceeds the permitted value the designer should either revise the design parameters (slab depth, prestress levels etc) or add additional bonded reinforcement and recalculate the cracked section, steel stresses and the resultant crack width, until compliance is achieved.

Crack width checks should be undertaken for the all 'design strips' where the allowable stresses are exceeded.

5.8.4 Deflection control

This is a Serviceability Limit State relating to the complete structure. The deflections of a structure, or of any parts of a structure, should not adversely affect appearance or performance.

Deflections that could damage adjacent parts of the structure (e.g. finishes) should be limited. The calculated deflection after construction (including the effects of creep and shrinkage, and camber) for the quasi-permanent loads should not normally exceed span/500 (see Eurocode 2, Clause 7.4.1).

The prediction of serviceability deflections is complex and requires a detailed knowledge of the likely loading regime and the age of the concrete at the time of loading. In addition the properties of the concrete and the stress levels within it have a major influence on the long-term deflections.

As a guide for a prestressed solid slab, continuous over two or more spans in each direction, the span/depth ratio should not generally exceed 42 for floors and 48 for roofs. These limits may be increased to 48 and 52, respectively, if detailed calculations show acceptable behaviour with regard to short- and long-term deflections, camber and vibration. Lower span/depth ratios will often apply to slabs with high live/dead load ratios. The span/depth ratios for waffle slabs should not generally exceed 35.

Where deflections are not considered to be critical to the performance of the structure it will be generally acceptable to factor the elastic deflection from a grillage or finite element analysis to take account of the long-term effects. For regular structural grids with uniform loadings where the equivalent load effects of the initial prestress are less than the dead load, the factors in Table 6 are considered to be acceptable.

Table 6: Factor taking account of long-term effects.

Loading	**Factor related to short-term elastic deflection value**
Dead	3.0
Post-tensioning (after losses)	3.0
Live	1.5

Note: The factor should be applied to the deflections obtained from an elastic analysis of the structure using the quasi-permanent load combination.

At design strips where the tensile stresses have exceeded the value in Table 5 the effects of cracking should be taken into account by modifying the *EI* properties of the concrete. For much proprietary software this may be done by modifying either the *I* or *E* value. It should be noted that the modification of the *E* value also effects the axial stress distribution and the axial deflections, although this is usually not significant.

The modification to *I* or *E* should be applied to all slab elements in the zone of the relevant Design Section and should encompass all slab elements within the line defining the point of contraflexure in the bending moment diagram. The modification factor to the *EI* value will vary depending on the extent of the cracking but will usually be in the range of 0.7–0.9.

A single iteration of the *EI* modification is considered acceptable and it should be undertaken, where required, prior to the application of factors for the long-term deflection prediction.

For structures where the deflections are considered to be more critical to the performance of the structure a detailed deflection analysis will be required. In such cases the following factors should be considered:

- age of concrete at time of loading
- concrete creep effects
- loading patterns and regime
- shrinkage curvature
- restraining effect of bonded reinforcement
- cracking in the concrete section.

5.8.5 Ultimate Limit State

An Ultimate Limit State (ULS) check in accordance with Eurocode 2, Section 6, is necessary on all floors in addition to the Serviceability Limit State analysis and design previously covered. In this condition, the factored dead and applied loads are considered together with the secondary effects of prestressing (see Section 5.6). The primary prestress effects are normally considered as part of the section strength. Additional un-tensioned reinforcement may be required in order to generate an adequate moment capacity.

The design tensile stress in unbonded tendons, f_{pb}, may be obtained from the equations given below. These have been developed from the results of tests in which the stress in the tendons and the length of the zone of inelasticity in the concrete were both determined.

$$f_{pb} = f_{pe} + 7000d\,(1 - 1.36 f_{pu} A_{ps} / (f_{ck}\, bd))/l$$

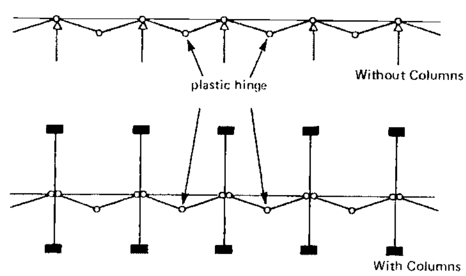
where

f_{pe} = design effective prestressing the tendons after all losses
d = effective depth to the centroid of the steel area A_{ps}
A_{ps} = area of the prestressing tendons in the tension zone
b = width or effective width of the section or flange in the compression zone
l = length of the inelastic zone = 10 × the neutral axis depth, x. (The floor is considered to develop both elastic and inelastic zones.)

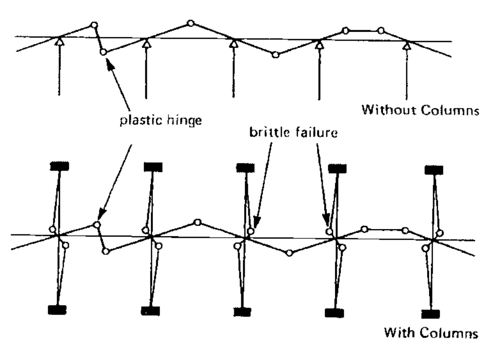
$$x = 1.98d\,\{(f_{pu} A_{ps})/(f_{ck}\, bd)\}(f_{pb} / f_{pu})$$

The extension of the concrete at the level of the tendons is assumed to be negligible in the elastic zones and the extension in the inelastic zone is assumed to be taken up uniformly over the length, *l*, of the tendon. This is discussed further in *The ultimate moment of resistance of unbonded prestressed concrete beams*[24] and *The ultimate moment of resistance of unbonded partially prestressed reinforced concrete beams* [25].

Hence, for a simply supported floor there is only one inelastic zone associated with the failure, but with a continuous floor the number of inelastic zones required for failure is more complex (see Figure 40).



a) Ductile failure



b) Brittle failure

Figure 40: Zones of inelasticity required for failure of a continuous member.

The length of tendon, l, can be modified, bearing in mind that if the tendon does not continue the full length of the continuous floor it may not include all the inelastic zones necessary for failure. It is therefore prudent to assume no more than one inelastic zone per span, and no more than two inelastic zones for the full length.

Flat slabs (two-way spanning on discrete column supports)

The Design Sections defined in Section 2 may be used for the checks at ULS. It should however be noted that at ULS, due to cracking an averaging of the bending moments across the full panel would normally produce an acceptable solution.

Where an evenly distributed tendon layout is used, some redistribution of moments away from the column strips is required. Experience shows that for normal slabs under normal loads and with normal levels of prestress, sufficient ductility exists for this to take place. However, where slabs carry heavy or abnormal loads, where higher than normal levels of prestress are used, or where tendons are prestressed to lower than normal levels, careful consideration of the ability for the moments to redistribute is required. This consideration should note the following features particular to post-tensioned flat slabs:

- The large strain required to yield bonded tendons, which is offset by the prestress
- The nature of redistribution across the slab rather than from support to mid-span
- The different mechanisms for redistribution in slabs made with bonded and unbonded tendons.

5.8.6 Progressive collapse

Where progressive collapse involves the use of unbonded tendons in key elements, the maximum stress in the unbonded tendon should not exceed $0.85f_{pu}$. This ensures that the anchorages are not overstressed, and protects against failure of catenary action.

In unbonded members there is also the risk that if tendons are severed accidentally there will be a 'progression' of failure for the full length of the tendons. This is particularly relevant for one-way spanning members such as beams, ribs and slabs spanning onto beams or walls. In such cases where horizontal progressive collapse is of concern, it is necessary to add reinforcement. This should be provided to satisfy the accidental load (see EN 1990[26], Clause 6.4.3.3 and UK National Annex Table A1.3 (UK)), and reduced material factor in accordance with Eurocode 2, Clause 2.4.2.4 for 'effects of exceptional loads or localised damage'. Reinforcement should be provided in accordance with Eurocode 2, Clause 9.10.

Experimental and practical evidence in the USA has established that this problem does not occur in the internal bays of flat slabs due to the overall 'plate' or membrane action. The possibility of horizontal progressive collapse of edge and corner panels of flat slabs must be considered.

These panels should be supported for the situation where the tendons parallel to the edge have been severed. This support can typically be provided by bonded reinforcement in the panel or an edge beam.

5.8.7 Designed flexural un-tensioned reinforcement

Additional un-tensioned reinforcement should be designed to cater for the full tension force generated by the assumed flexural tensile stresses in the concrete (see Figure 41) for the following situations:

- all locations in one-way spanning floors using unbonded tendons
- all locations in one-way spanning floors using unbonded tendons where transfer tensile stress exceeds $0.75f_{ctm}$ (where f_{ci} replaces f_{ck}) and using bonded tendons where the transfer tensile stress exceeds $1.2f_{ctm}$
- support zones in all flat slabs (less area of unbonded tendons)
- span zones in flat slabs using unbonded tendons where the tensile stress exceeds $0.4f_{ctm}$ and using bonded tendons where the tensile stress exceeds $1.2f_{ctm}$.

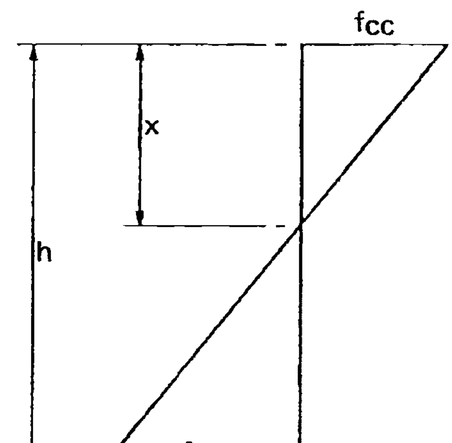


Figure 41: Section stresses used for the calculation of untensioned reinforcement.

The reinforcement should be designed to act at a stress of $(5/8)f_y$ as follows:

$$h - x = -f_{ct}\, h\, /(f_{cc} - f_{ct})$$

The value of f_{ct} is negative in tension

$$A_s = F_t\, /(5f_y\, /8)$$

where

$$F_t = -f_{ct}\, b\, (h - x)\, /\, 2$$

The reinforcement should be designed for the stresses at Serviceability Limit State, both after all prestress losses and at transfer conditions. It should be placed in the tensile zone, as near as practicable to the outer fibre. At transfer any designed reinforcement is likely to be on the opposite face to that required after all losses.

At ULS additional un-tensioned reinforcement may also be required (see Section 5.8.5). Any reinforcement provided for the SLS may also be used in the calculation of the moment capacity at ULS.

The designed reinforcement should be checked against the minimum requirements given in Section 5.8.8.

5.8.8 Minimum un-tensioned reinforcement

Where fire ratings of greater than 2 hours are required, it is recommended that anti-spalling reinforcement be placed in the soffit when no other reinforcement is provided.

One-way spanning floors

Bonded tendons:
There are no minimum un-tensioned reinforcement requirements for one-way spanning floors with bonded tendons. It is considered that these floors have sufficient tendon-to-concrete bond to distribute flexural cracking. Care should be taken to ensure sufficient reinforcement is provided to guard against cracking before stressing, if early phased stressing is not employed.

Unbonded tendons:
One-way spanning floors with unbonded tendons should have minimum reinforcement in accordance with Eurocode 2, Section 9. This reinforcement should be spread evenly across the full width of the slab in accordance with the spacing rules given in Eurocode 2, Sections 8 and 9.

Flat slabs (two-way spanning on discrete column supports)

All flat slabs should have minimum un-tensioned reinforcement at column positions to distribute cracking. The cross-sectional area of such reinforcement should be at least 0.075% of the gross concrete cross-section ($0.00075A_c$), and should be concentrated between lines that are 1.5 times the slab depth either side of the width of the column. The reinforcement should be placed as near as practicable to the top of the floor, with due regard for cover and tendon location, and should extend at least $0.2 \times L$ into the span or as far as necessary by calculation (see Sections 5.8.1 and 5.8.2). The maximum pitch of the reinforcement should be 300mm.

In the span zone, there are no minimum requirements. However, when unbonded tendons are used it would normally be necessary to provide designed un-tensioned reinforcement in the bottom of the slab (see Section 5.8.1). This reinforcement should extend at least to within a distance of $0.2 \times L$ (plus an anchorage length), measured from the centre of the support. It should be placed at a spacing of 3 × slab thickness or 500mm, whichever is the lesser.

Minimum reinforcement (or bonded tendons) of 0.1% should be provided in the hogging regions of a slab. The spacing of this should not exceed 500mm.

Slab edges

Un-tensioned reinforcement should be placed along edges of all slabs. This should include U-bars laced with at least two longitudinal bars top and bottom, as shown in Figure 42 (See also Section 5.11). Reinforcement should be provided in the triangular unstressed area between anchorages (See Section 5.12).

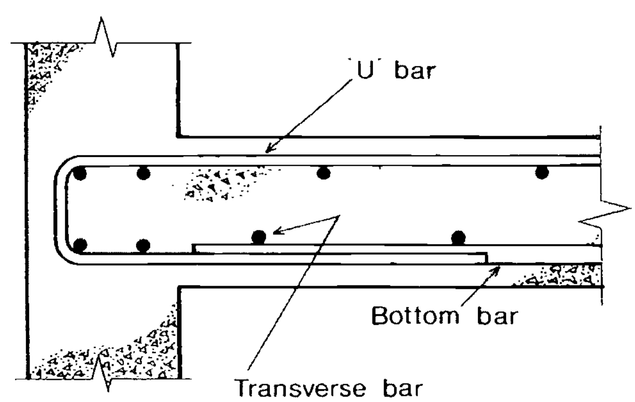


Figure 42: Reinforcement layout at the edge of a slab.

5.9 SHEAR STRENGTH

5.9.1 General

The shear capacity of prestressed elements is made up from three components.

1. The concrete shear component (or in the case of a shear reinforced flat slab the concrete and shear steel component): $V_{Rd,c}$ or $V_{Rd,cs}$
2. The part of the shear that is carried by arch (vault for flat slabs) action not dependent on bonded reinforcement.
3. The part of the load which does not act on the failure surface as it is carried to the columns by the vertical component of the tendons: V_p.

When calculating the contribution of the prestressing force at ULS, both the direct stress, σ_{cp}, and the beneficial effects due to the vertical component of the prestress force, the mean value of prestress calculated, should be multiplied by an appropriate safety factor γ_p. The value of γ_p, given in UK National Annex, is 0.9 when the prestress effect is favourable, and 1.1 when it is unfavourable.

5.9.2 Beams and one-way spanning slabs

Beams and one-way spanning slabs should generally be analysed in accordance with Eurocode 2, Clause 6.2. The following additional notes are intended to guide the user in the application of the rules.

In the case of linear elements requiring shear reinforcement the contribution of the concrete to the shear strength is ignored and calculation is based on the variable strut method. The effect of levels of prestress up to 25% of the design compressive strength is to increase the value of $V_{Rd,max}$ (the maximum strut force) that can be used. This allows higher values of Cotθ (the strut angle) to be used which in turn reduces the number of links required to take a given shear force. The result of this approach is that to obtain benefits from the axial compression the designer will need to maximise the value of Cotθ. The contribution of the tendon inclination can be included with the term V_{td}, which is the design value of the shear component in an inclined tensile chord.

For one-way slabs that do not include shear reinforcement, the contribution of the concrete is calculated including a σ_{cp} term that allows for the beneficial effects of axial prestress. Eurocode 2 is unclear as to whether the vertical component of the tendon force can be included in this situation. However it is recommended here that V_{Ed} (the design shear force) is reduced by the vertical component of the prestress.

5.9.3 Flat slabs (punching shear)

The approach taken to checking punching shear in Eurocode 2 is to check the shear at the column face and at a perimeter of $2d$ (where d is the effective depth of the tension steel) from the column face. If shear reinforcement is required a further check to ascertain its extent is made. The basic equations for shear capacity at $2d$ from the column are discussed below.

Concrete component:

$$v_{Rd,c} = \frac{0.18}{\gamma_c}\ k\ (100\rho_l\ f_{ck})^{1/3}\ +\ 0.10\sigma_{cp}$$

where:

f_{ck} is in MPa

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \qquad d \text{ in mm}$$

$$\rho_l = \sqrt{\rho_{ly}\ \cdot\ \rho_{lz}} \leq 0.02$$

ρ_{ly}, ρ_{lz} relate to the bonded tension steel in y- and z-directions respectively.

The values ρ_{ly} and ρ_{lz} should be calculated as mean values taking into account a slab width equal to the column width plus $3d$ each side or the actual dimension if less. Such reinforcement should anchored beyond the control perimeter being considered.

When bonded tendons are present they may be included in the calculation of ρ_l. If the calculation of $v_{Rd,c}$ includes the contribution of the bonded tendons in the section then $V_{Rd,c}$ should be based on the weighted average d. Thus $V_{Rd,c} = v_{Rd,c}\ u\ d'$ where d' = the weighted (based on steel area) average effective depth of the reinforcing and bonded prestressing steel. This approach is consistent with the approach to beams with more than one level of steel. The calculation of d for each tendon as it crosses the shear perimeter is time consuming and where the prestressing tendon is below a certain level its contribution to the concrete strength is negligible. For this reason it is simpler to ignore the contribution of bonded tendons for normal design but where they are included a rigorous calculation of d' is essential.

$$\sigma_{cp} = (\sigma_{cy} + \sigma_{cz})/2$$

where

σ_{cy}, σ_{cz} = concrete stresses in the critical section in y- and z-directions (MPa, positive if compression):

$$\sigma_{cy} = \frac{N_{Ed,y}}{A_{cy}} \text{ and } \sigma_{cz} = \frac{N_{Ed,z}}{A_{cz}}$$

σ_{cp} represents the axial stress in the concrete and has the effect of increasing the concrete shear capacity $V_{Rd,c}$. As the $v_{Rd,c}$ term is then multiplied by the shear perimeter and the effective depth to give a total resistance, its contribution increases for outer shear perimeters. This implies that at outer perimeters the effect of the axial stress carries a larger part of the load. The contribution of the axial stress can be considered as enabling a set of vaults to occur that carry part of the shear load back to the column. It is difficult to imagine these vaults carrying greater amounts of shear further away from the column where they are flatter than closer to the column where they are steeper. As most of the calibration of the Eurocode 2 formula was carried out on the first control perimeter it is probable that the 0.10 factor should be reduced for perimeters further out. However for ease of calculation it is recommended that the $0.1\sigma_{cp}\ ud$ term should remain constant for subsequent perimeters, i.e. for subsequent perimeters the effect of σ_{cp} on $v_{Rd,c}$ is ignored and the applied shear force is reduced by $0.1\sigma_{cp}\ u_1 d$ where u_1 is the length of the control perimeter $2d$ from the column. In all cases the calculation of σ_{cp} should include the appropriate γ_p value as discussed in Section 5.9.1.

The value of σ_{cp} is calculated as an average of the two orthogonal directions whereas the length of the shear perimeter in the two directions and the magnitude of prestress could be significantly different. Where this is the case it is more appropriate to consider:

$$\sigma_{cp}u_1 = (\sigma_{cy}u_{zz} + \sigma_{cz}u_{yy})$$

where

u_{zz} = total length of perimeter in a direction generally parallel to the Z axis

u_{yy} = total length of perimeter in a direction generally parallel to the Y axis (See Figure 43)

N_{Edy}, N_{Edz} = longitudinal forces across the full bay for internal columns and the longitudinal force across the control section for edge columns. The force may be from a load or prestressing action

A_c = area of concrete according to the definition of N_{Ed}

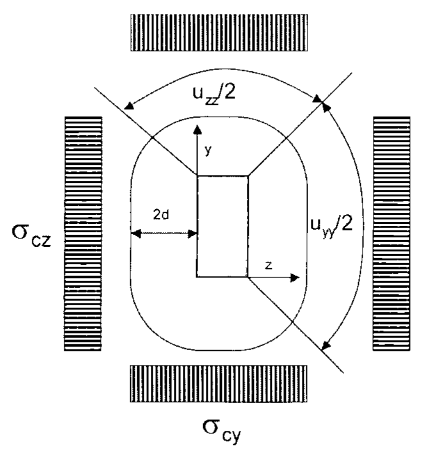


Figure 43: Perimeter lengths.

When calculating the longitudinal forces it is important, for edge columns, that the locations of anchorages are taken into account. A 45° spread may be assumed but in certain cases this may lead to parts of a shear perimeter edge being unstressed, and/or parts of the perimeter being subject to significantly higher than average stresses. It is reasonable to average that part of the prestress force crossing part of the perimeter edge over the whole perimeter edge.

When reinforcement is present the shear resistance is:

$$v_{Rd,cs} = 0.75v_{Rd,c} + 1.5\ (d/s_r)\ A_{sw}.f_{ywd,ef}\,(1/(u_1 d))\ \sin\alpha$$

where

A_{sw} = area of shear reinforcement in each perimeter around the column

s_r = radial spacing of layers of shear reinforcement

α = angle between the shear reinforcement and the plane of the slab

$f_{ywd,ef}$ = effective design strength of the punching shear reinforcement
= $250 + 0.25d \le f_{ywd}$ (MPa)

d = mean effective depth of the slab reinforcement in the two directions (mm)

The vertical components of the tendons should also be allowed for by reducing or increasing the applied shear load by the vertical component of the tendon force, again factored by the appropriate value of γ_p. At columns where the effects are usually beneficial it is important to consider the anchorage of the tendon within the critical shear perimeter. Figure 44 shows the effect of the outer tendons pulling out of the column shear cone and these cannot take vertical load back to the column. For this reason only tendons passing within $0.5h$ of the column face should be considered and the angle of the tendon considered should be that at $0.5h$ from the column face. Simplistically this means that if a load balancing approach has been used, the total shear applied to the column can be reduced by the load applied by the tendons within $0.5h$ of the column. This value should remain constant for outer perimeters.

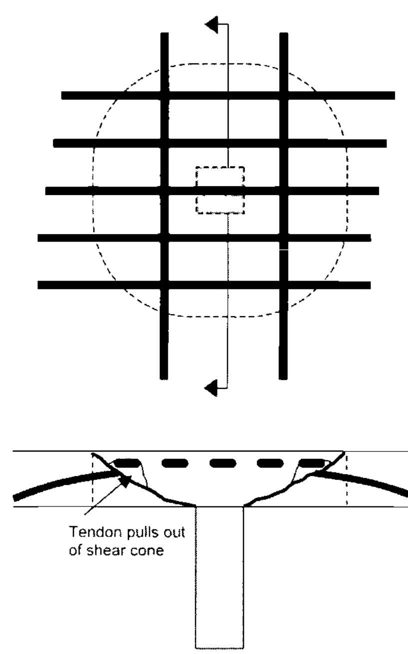


Figure 44: Catenary action of tendons at column head.

As the contribution of prestress $\sigma_{cp}u_1$ and the contribution of the vertical component of the prestressing strand are not dependent on the perimeter, it is more convenient from a design point of view to consider them on the resistance side when calculating the capacity of perimeters beyond u_1 and in particular for calculating the perimeter $u_{out,ef}$ (the control perimeter at which shear reinforcement is not required). Thus Expression 6.54 of Eurocode 2 becomes:

$u_{out,ef}$ = ($\beta\ V_{Ed}$ − $0.1\sigma_{cp}u_1 d$ − the balancing load from tendons within $0.5h$ of column face)/($v_{rd,c}d$)

where the calculation $v_{rd,c}$ does not include the $0.1\sigma_{cp}u_1$ term.

5.9.4 Structural steel shearheads

There are two typical types of structural steel shearhead.

The first type form a full perimeter larger than the column and the structural sections carry the entire load back to the column. This type of shearhead can be too deep for typical post-tensioned slab applications and can also interfere with the tendon profile. For these reasons this type is not commonly used.

The second type is a cruciform layout. The load is transferred to the column by a combination of the structural steel section and shear in the remaining concrete (Figure 45). This requires compatibility between the stiffness of the shearhead and the concrete. The effect is not adequately covered in either British or European codes and so design of these elements should be based on ACI 318[27]. It should be noted that the ACI approach uses different load factors to those of Eurocode 2 and strength reduction factors instead of material factors. For this reason it is recommended that, for the punching shear design, a separate load rundown is carried out and that the design follows the strength reduction approach consistently. It should also be noted that ACI 318 contains no reduction for concrete shear capacity with increasing member depth, and the tests that formed the basis of this approach were on 146mm deep specimens. For these reasons the method is not recommended for slabs deeper than 300mm.

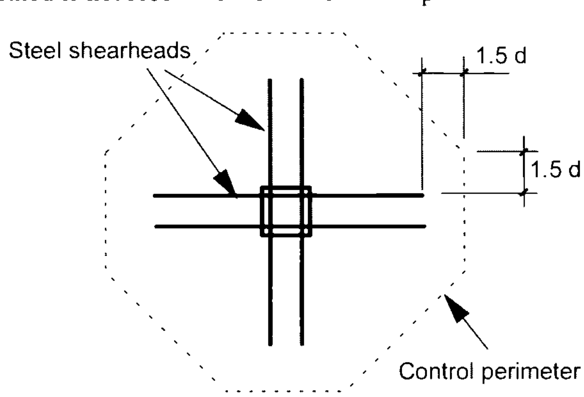


Figure 45: Structural steel shearhead.

In order to ensure compatibility with Eurocode 2 a control perimeter outside the ends of the shearhead as shown in Figure 45 should be checked. For this check the full perimeter may be used.

5.10 OPENINGS IN SLABS

Tendons should be continuous and displaced horizontally to avoid small openings. If tendons are terminated at the edges of large openings, such as at stairwells, an analysis should be made to ensure sufficient strength and proper behaviour. Edges around openings may be reinforced similarly to conventionally reinforced slabs; in the case of large openings, supplementary post-tensioning tendons may be used to strengthen the edges around openings.

5.11 ANCHORAGE BURSTING REINFORCEMENT

Reinforcement is usually required to resist the tensile stresses caused by the concentration of the forces applied at the anchors. At some distance from the edge of the floor (or the anchorages) it can be assumed that the distribution of stresses is the classic linear distribution and depends only on the magnitude and position of the resultant of the forces applied to the edge of the floor.

Between the edge and the above plane the lines of force are curved and give rise to transverse tensile stresses in both directions perpendicular to the applied force direction.

Where groups of anchorages exist, as is often the case for 'banded' slab tendons, the bursting stress zones for both the individual and collective anchorages should be considered, and reinforcement placed accordingly.

Care should also be taken to ensure that the phasing of the application of prestress to anchorage groups does not create a bursting condition that may be critical. If this condition is unavoidable, reinforcement should be added accordingly.

Eurocode 2, Clause 6.5.3 provides a method of designing the required transverse reinforcement.

Two examples showing the calculation and detailing of bursting reinforcement are given in Appendix E.

5.11.1 Serviceability Limit State (SLS)

At the SLS the design bursting tensile force, F_{bst} (given as 'T' in Expressions 6.58 and 6.59 of Eurocode 2) in an individual square end block loaded by a symmetrically placed square bearing plate, may be calculated based on the tendon jacking force, P_0 (given as 'F' in Expressions 6.58 and 6.59 of Eurocode 2). With rectangular anchorages and/or rectangular end blocks, the bursting tensile forces in the two principal directions should be assessed in each direction. When considering the horizontal plane of the slab, b should be taken as the spacing of anchorages and $h = b$. When considering the vertical plane of the slab, b should be taken as the depth of slab.

The force F_{bst} is distributed in a region extending h from the loaded face, and should be resisted by reinforcement in the form of spirals or closed links, uniformly distributed throughout this region, and acting at a stress of 200MPa.

When a large block contains several anchorages it should be divided into a series of symmetrically-loaded prisms and each prism treated in the above manner. However, additional reinforcement will be required around the general cross-section of the beam. Also when anchorages occur within the plan area of the floor rather than at the perimeter, it may be necessary to include 'following' reinforcement. This reinforcement runs parallel to the tendon past the anchorage to limit cracking adjacent to the anchorage. These effects are discussed in CIRIA Guide No. 1[28].

5.11.2 Ultimate Limit State (ULS)

For members with unbonded tendons the design bursting tensile force, F_{bst}, should be assessed on the basis of the characteristic tendon force; the reinforcement provided to sustain this force may be assumed to be acting at its design strength ($0.87f_y$). No such check is necessary in the case of members with bonded tendons.

Post-tensioning system suppliers often test their anchorage systems in concrete prisms, reinforced in a similar manner to that encountered in practice and using a prism size similar to the common on-site member thickness etc. Such tests may be deemed to satisfactorily model the on-site conditions and the reinforcement may be considered adequate provided suitable safety factors are employed.

5.12 REINFORCEMENT BETWEEN TENDON ANCHORAGES

Figure 46 shows an area of slab between tendon anchorages that requires reinforcement to span the unstressed zones. Any prestressed tendons that pass through this zone, parallel to the slab edge, may be included with the relevant reinforcement, provided it is in the local tension zone.

The area of tension reinforcement (and/or prestressed tendons) provided parallel to the slab edge should resist bending moments from the ultimate vertical loads calculated for a continuous slab spanning l_a. This reinforcement should be evenly distributed across a width equal to $0.7l_a$, and should be continuous along the edge.

The area of reinforcement placed perpendicular to the slab edge should be the greater of 0.13% bh, or a quarter of the reinforcement provided parallel to the edge. It should be placed evenly between anchorages, and extend the greater of l_a or $0.7l_a$ plus a full anchorage length into the slab.

5.13 VIBRATION

Prestressed floors are usually thinner or span further than reinforced floors. They therefore tend to have lower natural frequencies and greater consideration must be given to their dynamic performance.

The values given in Table 1 have been shown to give floors whose vibration performance is generally acceptable. For floors not satisfying the requirements of Table 1 or where there is a specific requirement for vibration performance due to building use (e.g. laboratories) or due to client specification, reference should be made to Appendix G.

5.14 LIGHTWEIGHT AGGREGATE CONCRETE

Additional considerations on the use of lightweight aggregate concrete are given in Eurocode 2, Section 11, and the *Guide to the structural use of lightweight aggregate concrete*[29].

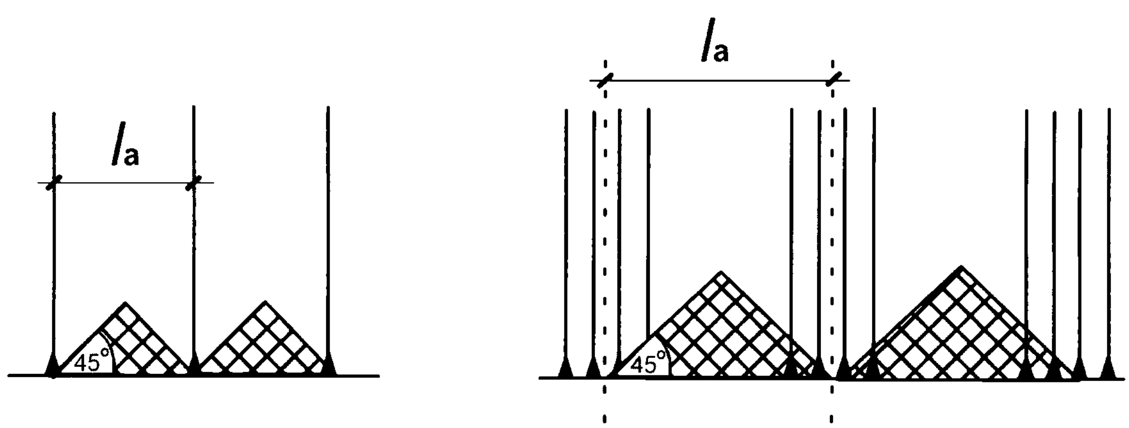


Figure 46: Unstressed areas of slab edges between tendons requiring reinforcement.

6 DETAILING

Reference should be made to *Standard method of detailing structural concrete*[30].

6.1 COVER TO REINFORCEMENT

Cover is dependent on durability requirements and fire resistance, whichever condition is the more onerous. Consideration should be given to the layout of both tendons and un-tensioned reinforcement when deciding the critical cover requirements.

6.1.1 Bonded tendons

The cover to the tendons should be in accordance with the requirements for prestressed concrete in Eurocode 2 Part 1.1, Clause 4.4.1.2. The cover is measured to the outside of the duct. It should be noted that the cover to the centre of the tendon will be more than that to the centre of the duct, since the tendon will press against the wall of the duct. Typically for a flat slab, the minimum required cover to the oval duct (with four No. 15.2mm diameter strands) is 35mm. The nominal cover, c_{nom}, is thus 35 + (allowance in design for deviation, Δc_{dev}). Unless special measures are taken to ensure reliable quality (see Eurocode2 Part 1.1, Clause 4.4.1.3) the value of Δc_{dev} should be taken as 10mm. Hence c_{nom} = 45mm.

The fire requirements for cover and member size should be in accordance with Eurocode 2 Part 1.2. Clause 5.2 (5) notes that the cover for prestressing strand should be 15mm more than that given in the tables for reinforced and prestressed concrete. Typically for a flat slab with a 1½ hour fire rating the required axis distance (distance from the centre of strand to the concrete surface) is 25 + 15 = 40mm.

6.1.2 Unbonded tendons

Eurocode 2 Part 1.1 requires that the cover should be in accordance with a European Technical specification. The tendon is normally specified as a nominal diameter (e.g. 12.9 or 15.7mm for 7-wire super strand): 3mm should be added to the diameter to allow for the thickness of sheathing. Typically for a flat slab, the minimum required cover to the sheath is 20 for buildings and 35 for car parks. The nominal cover, c_{nom}, is thus 30mm and 45mm, respectively.

The fire requirements for cover and member size should be in accordance with EC2 Part 1.2 and are similar to those for bonded tendons.

6.1.3 Un-tensioned reinforcement

The cover to the reinforcement should be in accordance with the requirements for prestressed concrete in Eurocode 2 Part 1.1, Clause 4.4.1.2. Typically for a flat slab, the minimum required cover is 20 for buildings and 35 for car parks. The nominal cover, c_{nom}, is thus 30mm and 45mm, respectively.

The fire requirements for cover and member size should be in accordance with Eurocode 2 Part 1.2. Typically for a flat slab with a 1½ hour fire rating the required axis distance (distance from the centre of bar to the concrete surface) is 25mm.

6.1.4 Anchorages

Eurocode 2 Part 1.1 requires that the cover to anchorages should be in accordance with a European Technical specification.

6.2 TENDON DISTRIBUTION

Various methods for distributing the tendons can be used. These are discussed in Section 2.4. From a construction aspect it is normal to lay the tendons banded in one direction and evenly distributed in the other, which minimises the amount of weaving and hence simplifies the laying procedure.

At the column/floor intersection, where there is no shear reinforcement, it is recommended that a minimum of two tendons should pass within $0.5h$ of the column face for internal columns and a minimum of one tendon for external columns parallel to the slab edge. Only tendons within this distance are permitted to contribute towards the shear resistance (see Section 5.9.3). Where shear reinforcement is present the $0.5h$ limit may be increased to h.

For ribbed slabs or beams, the distributing of tendons is dictated by the spacing of members but generally one should endeavour to have a tendon passing within $0.5h$ of the column face (see Figure 47).

For situations where it is not practically possible to place the prestressing tendons within $0.5h$ from the column, reinforcement should be placed to bridge the vertical force from the adjacent tendon to the columns as shown in Figure 48. The reinforcement should:

- be placed under the prestressing tendon
- have sufficient area to transmit the vertical component of prestressing for that tendon to the column
- extend a full anchorage length beyond the tendon
- lie within 0.5*h* of the column and at least one bar should pass over the column.

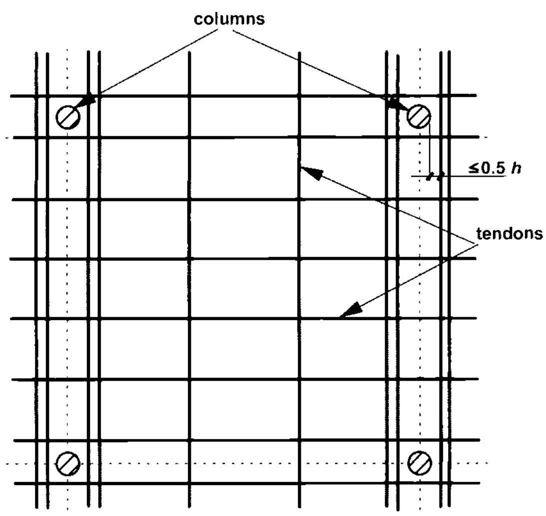


Figure 47: Position of tendons relative to columns.

6.3 TENDON SPACING

For suspended post-tensioned slabs the maximum spacing of uniformly distributing tendons should not exceed 6 × the slab depth for unbonded tendons or 8 × the slab depth for bonded tendons. Where the banded option is used both ways then the maximum spacing between tendons should be restricted to 10 × the slab depth for bonded tendons. Greater spacing may be used where it can be shown that the slab is capable of spanning between the tendons and supporting the applied design loads.

Unbonded tendons may be placed in groups if required. It is recommended that grouped tendons are laid side by side and do not exceed four tendons per group.

The minimum horizontal distance between ducts or groups of tendons should be the greater of 75mm or the group/duct width.

Should it be necessary to arrange the tendons in vertical layers in beams or ribs, then it is recommended that the gap between the layers should be at least the vertical dimension of the tendon or duct. In the case of bonded tendons where oval metal ducts are used, it is recommended that their positions are staggered to ease the placing of concrete.

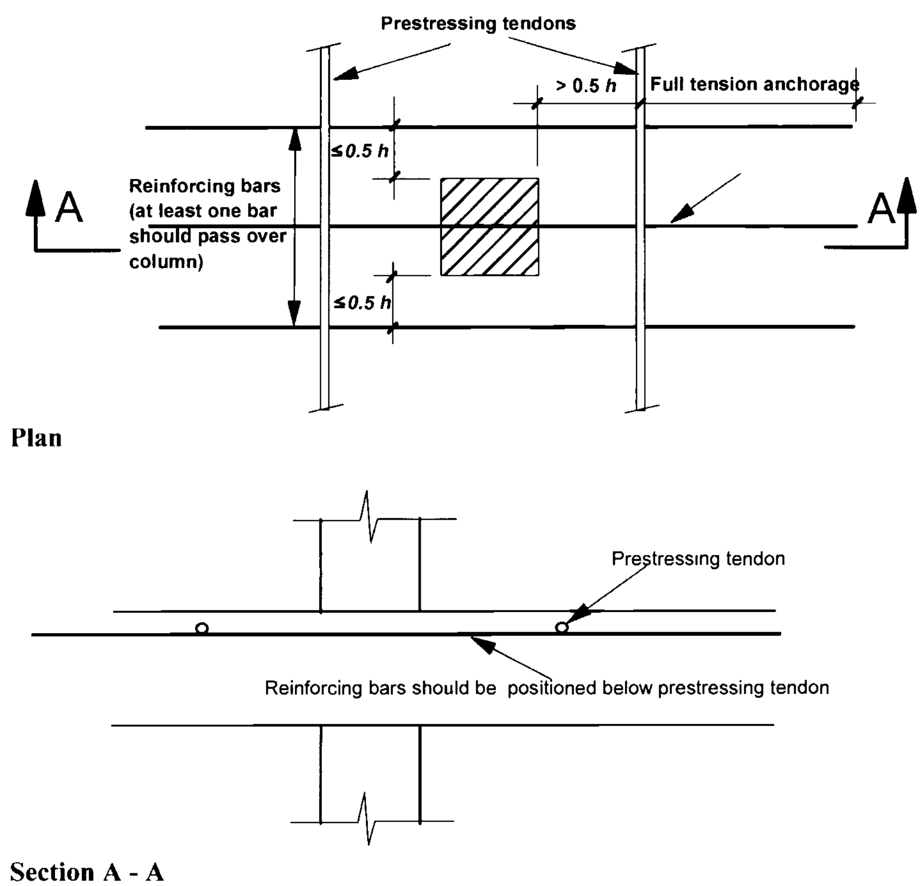


Figure 48: Additional reinforcement required where tendons are not within 0.5*h* from the column.

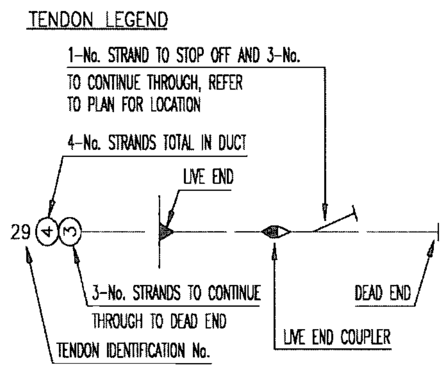
If tolerances on tendon positions are not stated, the values in Table 7 should be adopted.

Table 7: Tolerances on tendon positioning.

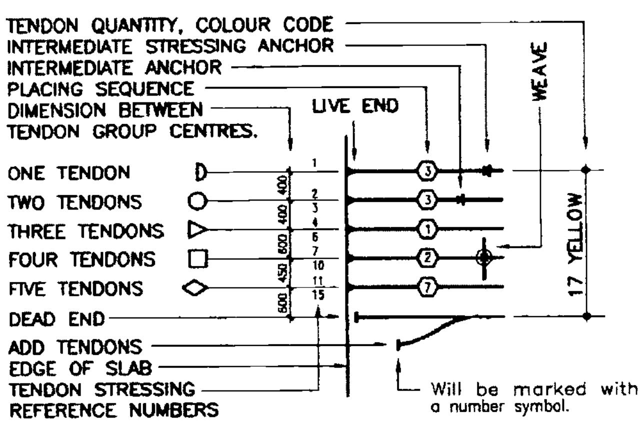
Slab thickness	Tolerances	
	Vertically	Horizontally
$h \leq 200$mm	$\pm h/40$	± 100mm
$h > 200$mm	± 5mm	± 100mm

6.4 TENDON NOTATION

Typical notation of tendons on drawings is shown in Figure 49. It is recommended that this legend figure is included on all tendon layout drawings.



a) Typical notation for use on tendon layout drawings

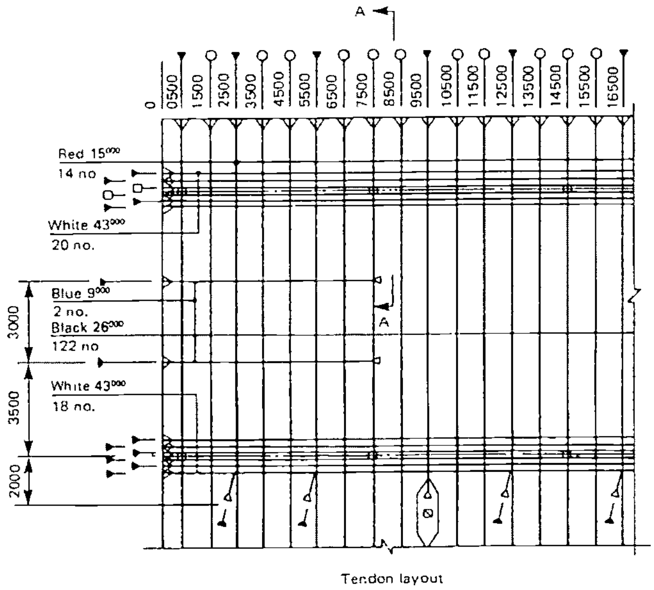


NOTE

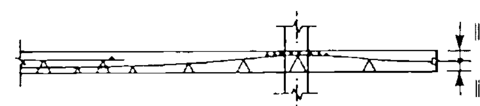
When more than one symbol appears on a tendon group, the number of tendons will equal the sum of the symbols.

b) Typical method for unbonded tendons

Figure 49: Typical notation for use on tendon layout drawings.

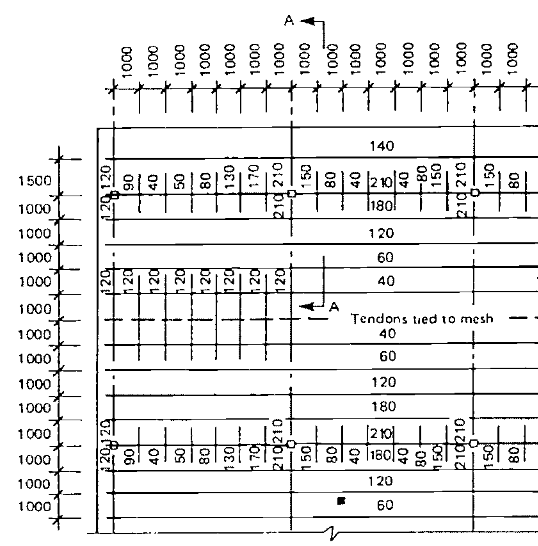


Placing sequence not shown



Section A.A

a) Flat slab tendon layout



b) Typical tendon profile and support layout for slab

Notes:
1. Height given is from soffit of slab to underside of tendon.
2. Diameter of support bar is 10mm.

Figure 50: Flat slab tendon and support layout detailing.

Figure 50a shows an example using the legend showing groups of tendons and anchorage types, together with the tendon sequence, detailed. This figure is taken from *Standard method of detailing structural concrete*[30].

Tendon profiles in the longitudinal and transverse directions are shown using an exaggerated scale for the vertical dimensions. These are usually given from the soffit of the slab to the centre line of the duct/sheath and are plotted at intervals of 1m. Closer centres may be necessary for sharp vertical curves. For ease of placement on site, shop drawings are detailed giving the vertical tendon position from soffit to underside of tendon.

6.5 TENDON SUPPORTS

The profile of the tendons is critical to the floor performance. It is therefore recommended that the support centres do not exceed 1m. For ribbed slabs or beams, support bars can be adequately held by firm wire ties. Welding is not recommended and should be only used with extreme care.

Figure 50b shows a typical support bar layout. The actual layout may be modified by the contractor depending on the support system adopted, so that the specified tendon profiles are attained and adequate support is provided.

6.6 LAYOUT OF UN-TENSIONED REINFORCEMENT

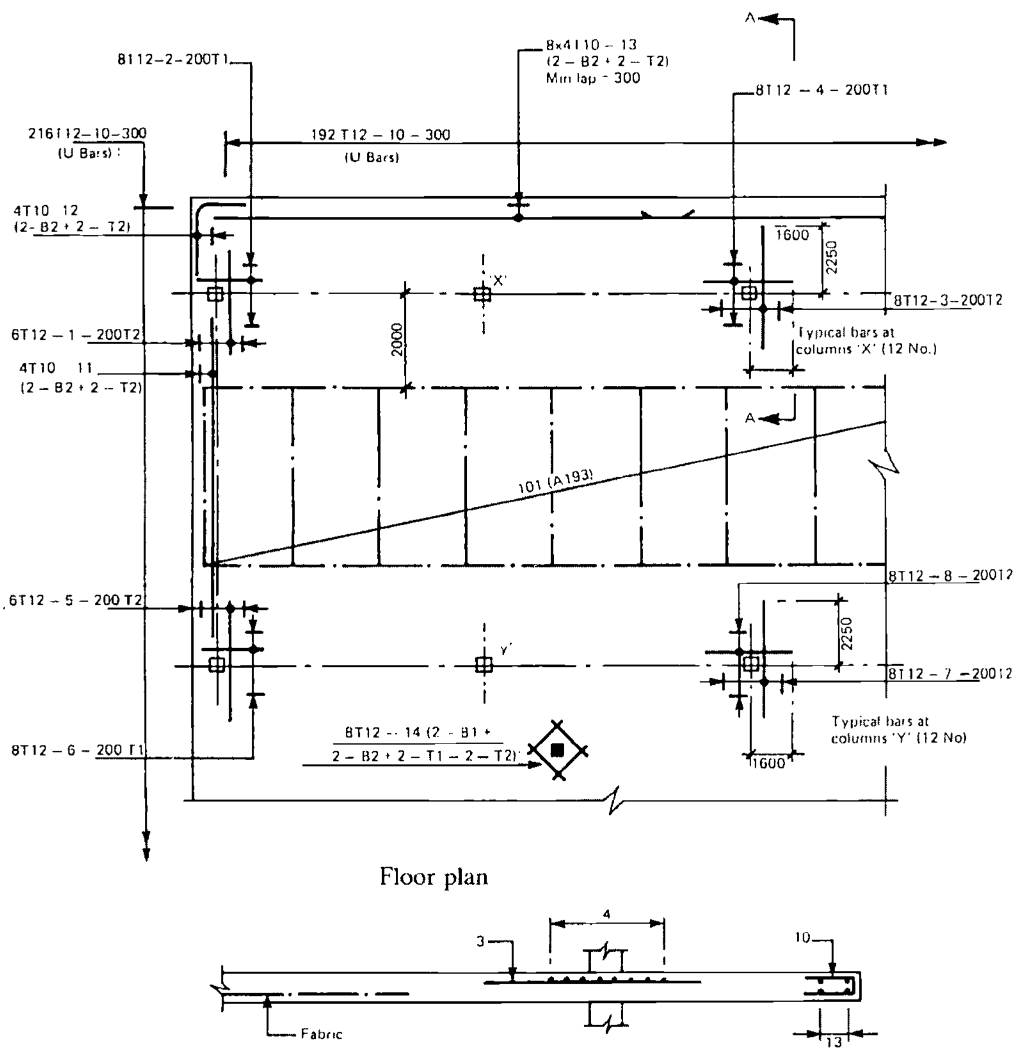
Figure 51 shows an example of the reinforcement that is always required at edges and in the top of flat slabs at columns. It also shows the reinforcement needed in the bottom of the slab at mid-span for some design applications. See Section 5.8.8 for details.

6.6.1 At columns

Reinforcement should be placed in the top of the slab over columns. The design of such reinforcement is described in Section 5.8.7 with minimum requirements given in Section 5.8.8. Figure 22 shows a typical arrangement of tendons and un-tensioned reinforcement around a column.

6.6.2 Shear reinforcement

Shear reinforcement in flat slabs, if required, is usually in the form of links or hairpins, although prefabricated shear reinforcement is available. Fabricated steel shearheads may also be used (see Figure 52 and Section 5.9.4 for details).



Floor plan

Section A.A showing reinforcement details

Figure 51: Flat slab reinforcement layout.

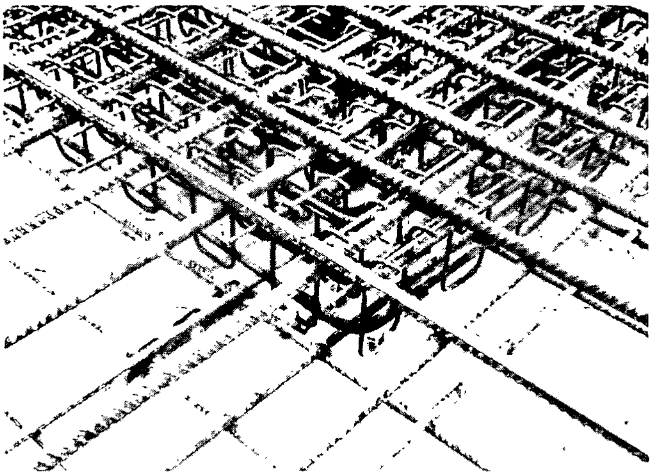


Figure 52: Prefabricated shear reinforcement.

6.6.3 At and between anchorages

An adequate amount of reinforcement should be placed at anchorage end blocks to avoid splitting of the concrete. A sample calculation to determine the amount of this reinforcement is given in Appendix F.

Reinforcement should be provided in the 45° wedge area between the anchorages (see Figure 46).

6.7 PENETRATIONS AND OPENINGS IN FLOORS

Unbonded tendons may be diverted around the openings as they are relatively flexible (see Figure 53). The change of direction of the tendon should occur away from the opening, and trimmer bars should be provided to avoid possible cracking at the corners.

The oval sheathing used in bonded tendons is rigid in the transverse direction, and only limited deviations can be made around openings. Openings ideally should be located between tendons, however tendons can be terminated either side of larger openings.

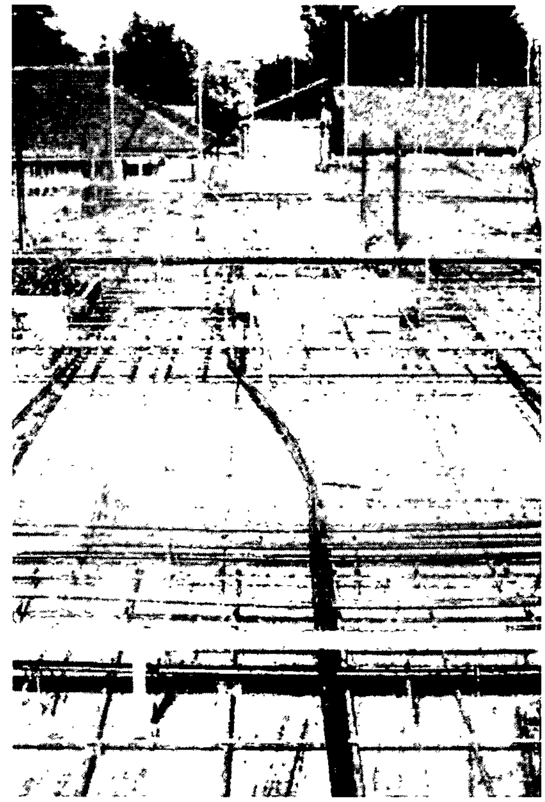


Figure 53: Unbonded tendons diverted around an opening.

Any penetrations made to a floor after it has been cast must be documented, included in the design and co-ordinated with the tendon layout.

The cutting of penetrations in finished slabs is not a problem in ribbed slabs where the tendon positions are, in effect, defined. Grouted tendons, providing the grout is effective, can be cut and with the loss of prestress over a transmission length from the cut.

Where unbonded tendons have been used, care must be taken to locate the tendons before any concrete is removed. Tendons can be cut and reinstated but it is recommended that this work be carried out by a specialist.

7 CONSTRUCTION DETAILS

7.1 SUPPLY AND INSTALLATION OF POST-TENSIONING SYSTEMS

It is assumed within the context of this design handbook that the supply and installation of post-tensioning systems is carried out in accordance with the National Structural Concrete Specification (NSCS)[15] and the UK Certification Authority for Reinforcing Steel (CARES) Certification Scheme or similar approved. The installation of such systems is assumed to be in accordance with CARES Appendix PT 2(a)ii or equivalent.

7.2 EXTENT OF POURS

The size of pour is limited by:

- the amount of early thermal shrinkage that will take place. This is linked with the amount of un-tensioned reinforcement provided.
- the ability to complete the pouring and finishing of the surface of the concrete.
- friction losses. With bonded tendons, these usually restrict the length of single-end stressed tendons to 35m, and double-end stressed to 65m for a slab and 25m and 50m, respectively, for a beam. The lower friction values for unbonded tendons extend these values to 45m and 100m, respectively, for slabs. Longer lengths are achievable but the friction losses should be carefully considered.

Prestressing tendons may be continuous through construction joints allowing larger areas without any permanent joints. Allowances should be made in accordance with good practice to accommodate temperature variations by the provision of expansion joints on larger slabs.

7.3 CONSTRUCTION JOINTS

Generally construction joints should be made in the vicinity of quarter and third points of the span from supports.

Shear provision in accordance with good practice should be made by the introduction of expanded mesh, by roughening the previously poured surface or by the introduction of a shear key.

There are generally two types of construction joints. The first is a coupled construction joint, where the post-tensioning tendons are 'coupled' together using a coupler. A coupled joint maintains the prestress in the concrete from the post-tensioning across the joint and post-tensioning is considered continuous through the joint.

In long slabs, intermediate anchorages may be introduced which allow the stressing to be continuous through the construction joint or the joints can be traditionally reinforced.

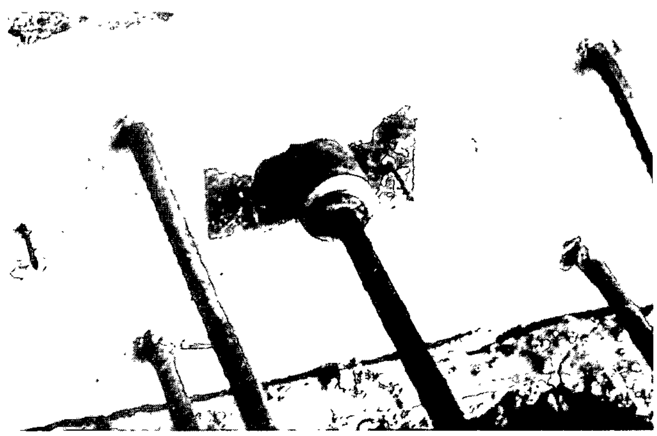
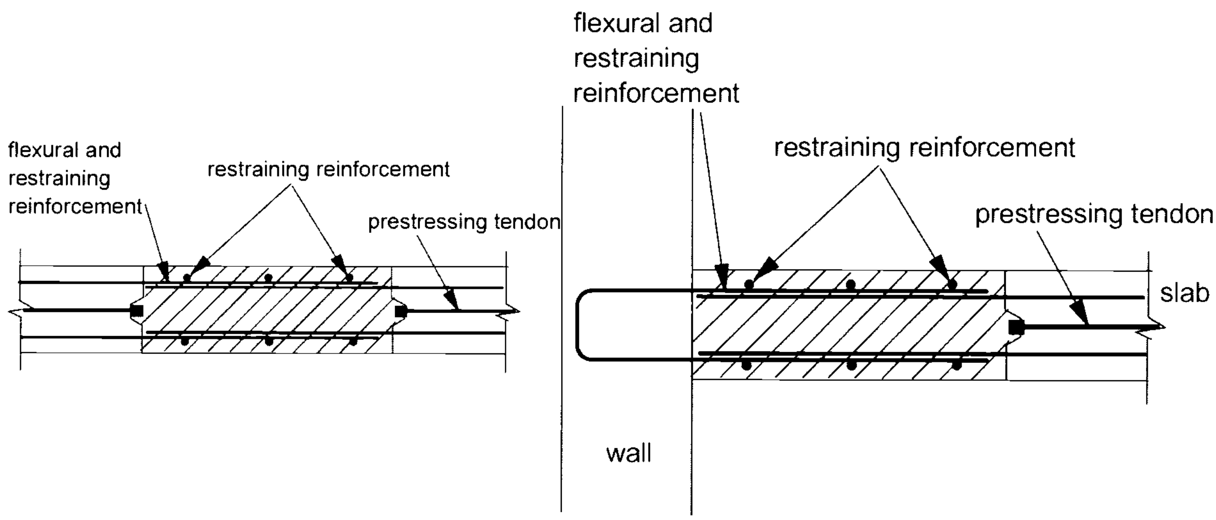


Figure 54: Intermediate anchor at construction joint.

The second type of construction joint is reinforced and not prestressed. Continuity is achieved through the lapped reinforcing bars. The quantity of reinforcement should be sufficient for strength and to prevent large cracks forming at the joint under the effects of temperature, creep and shrinkage.

For long slabs or slabs with stiff vertical elements permanent or temporary movement joints should be used to reduce the restraint to the post-tensioning force, creep and shrinkage. Figures 55 and 56 show examples of temporary closure strips. These strips are cast after stressing of the adjacent sections is complete. This operation should be delayed for as long a period as is reasonable to reduce the effects of creep and shrinkage.

In assessing the movement of slabs at expansion or contraction joints from the time of pouring concrete, a strain of 650×10^{-6} should be considered as normal. The drying out effect of air conditioning can increase this to 1000×10^{-6}.



a) Closure strip in slab

b) closure strip at junction of wall and slab

Figure 55: Typical release joints.

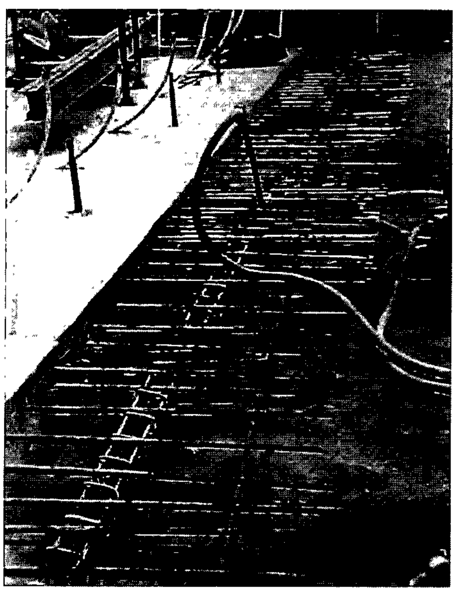


Figure 56: Infill strip.

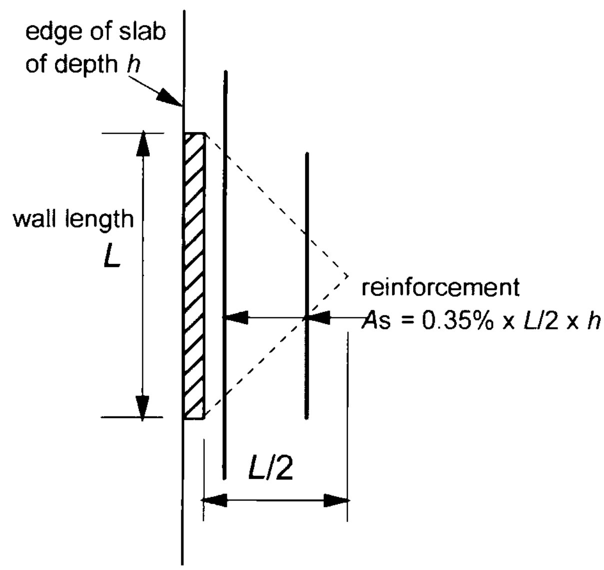


Figure 57: Distribution reinforcement close to restraining wall.

The use of dowels perpendicular to the direction of stressing should be avoided as this could prevent the stress being transferred to the slab. Generally dowels should be avoided in slabs stressed in two directions. Where there is restraint in the direction of prestressing (e.g. a shear wall) reinforcement should be placed parallel to the restraint to reduce and distribute any cracking as shown in Figure 57.

In areas where edge access is limited then the introduction of pans can be utilised. This requires a depression to be formed in the top of slab, which allows stressing of the live end from the surface of the slab see Figure 58.

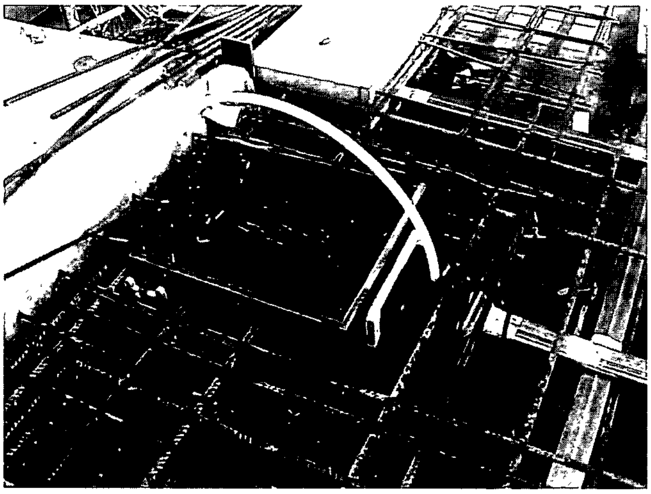


Figure 58: Intermediate anchorage.

7.4 PROTECTION OF ANCHORAGES

Tendons are normally anchored within the middle third of the slab to ensure adequate edge cover to the anchorage. Pocket formers at anchorages should be large enough to allow adequate trimming of the tendons after stressing, thus ensuring good end cover to the strand. Trimming should be carried out using a disc cutter or hydraulic shears (see Figures 59 and 60). In no circumstances should the tendon be trimmed by flame cutting. Pocket formers are normally proprietary plastic or polystyrene units that make up part of the anchorage fixings. Anchorages fixed to form work are shown in Figures 61 and 62. It is recommended for unbonded tendons that, after trimming the strands, the wedges and the strand end are coated with grease of similar specification to that used in the tendon and that a watertight cap be applied over the coated area. The minimum end cover to this cap should be 25mm.

Figure 59: Stand trimming using a disc cutter.

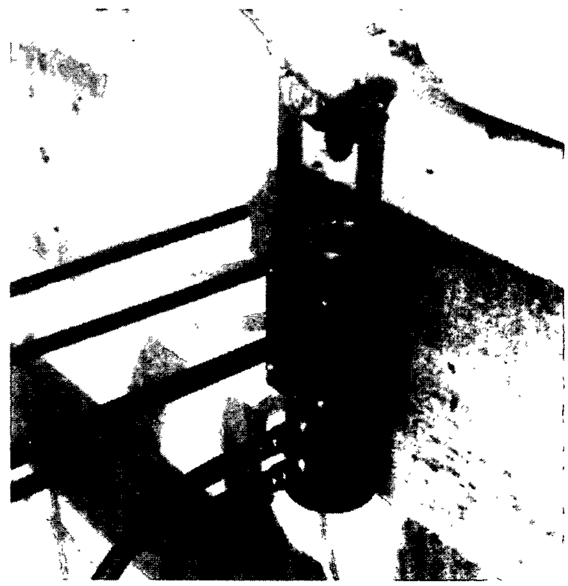


Figure 60: Stand trimming using purpose-made hydraulic shears.

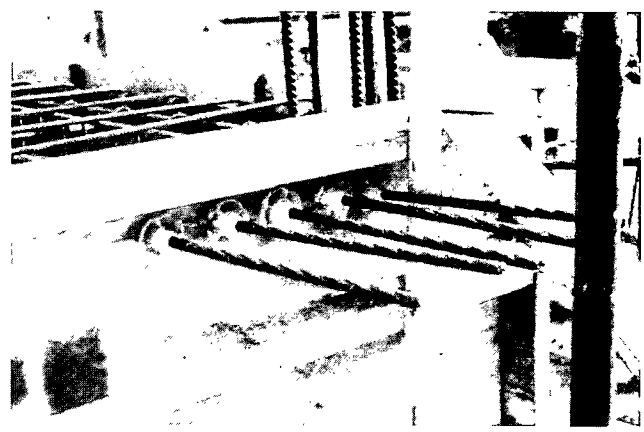


Figure 61: Anchorages for unbonded tendons: fixed to formwork.

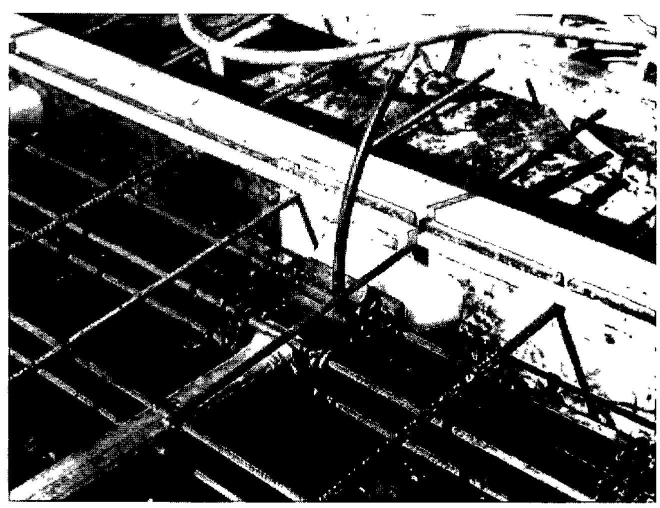


Figure 62: Anchorages for bonded tendons: fixed to formwork.

For bonded tendons after the strand has been trimmed then the anchor is treated with a resin sealer. The pockets for the anchorages are generally sealed with a sand cement dry pack. This has two functions. First it covers and protects the anchor and second it allows the air to be expelled during grouting. Where pockets are exposed to the elements or for aesthetic purposes then the dry pack is left low and a suitable non-shrink mortar is post-applied after grouting operations are complete (see Figure 63). In no circumstances should this mortar contain chlorides or other materials that could be harmful to the prestressing steel.

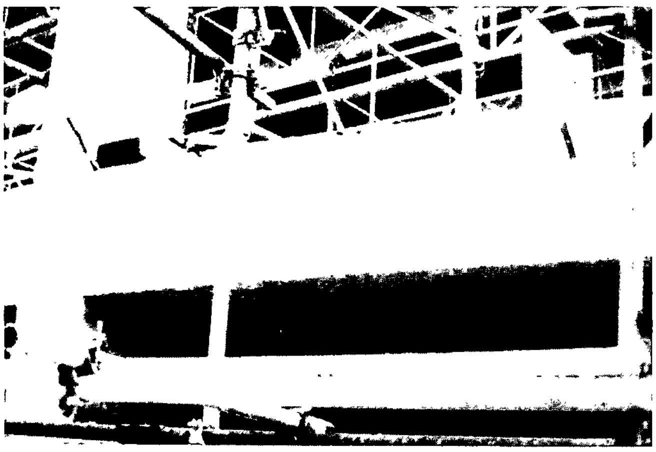


Figure 63: Anchorage blocks sealed with mortar.

7.5 BACK-PROPPING

Back-propping may be required to ensure that the construction loads can be safely carried by the earlier construction stages, and this must be considered by the designer in a similar manner to normal reinforced concrete construction. Stressing sequencing should also be considered as this may induce greater loads. When using propping to resist the uplift forces from carrying cantilever decks check that decks are not overstressed due to load balancing action acting with the upward load from the props.

7.6 STRESSING PROCEDURE

The stressing forces and sequence of stressing should be specified on the drawings. This has to be planned in such a way that the prestress is applied as uniformly as possible, and that no overloading of the formwork occurs. For systems with banded and uniformly distributed tendons, the banded tendons should normally be stressed first to ensure this is the case (see Figure 64). Wherever possible the use of different forces for tendons of the same size should be avoided.

It should be noted that after stressing a bonded system and before grouting has taken place, it should be considered as an unbonded system.

In members where early stressing is desired to reduce the risk of early shrinkage cracking, it is common to stress the tendons in two stages. The first stage is usually about 25% of the final prestress force, and is carried out as soon as the concrete has obtained adequate strength for the anchorage

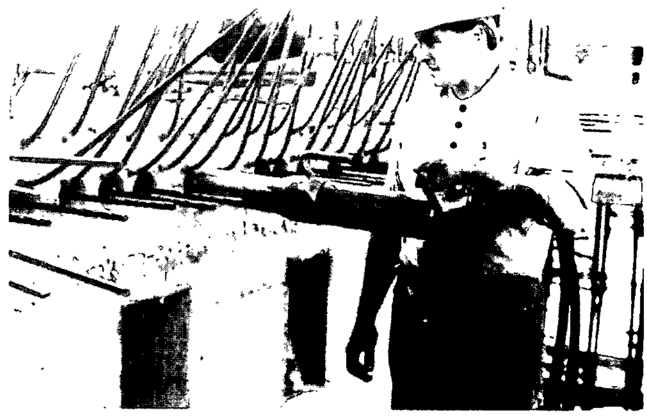


Figure 64: Stressing banded tendons at slab edges.

being used. This concrete strength is typically between 10 and 15MPa. It is important that sufficient site-cured cubes or cylinders are provided to determine the transfer strength (this is especially important in periods of cold weather).

Where a slab or system of secondary beams is stressed across primary beams attention must be given to the sequence of stressing in order to avoid damage to the formwork of the primary beams.

7.7 GROUTING

Grouting of the ducts for bonded tendons should carried out in accordance with the CARES specification for bonded and unbonded post-tensioned flat slabs[31] or equivalent.

7.8 SOFFIT MARKING

Tendon positions in flat slabs are not always apparent on completion of concreting. Recent practice has been to introduce soffit marking, where the cover to the tendon is less than the penetration of ceiling and service fittings. An illustration of typical marking is shown in Figure 65. Unpainted zones indicate no tendons. Dark zones indicate tendons near the soffit and white zones indicate tendons near the top of the slab.

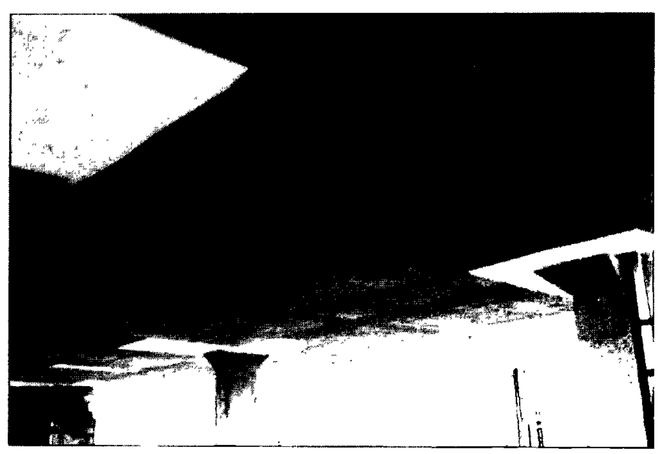


Figure 65: Soffit marking used to indicate tendon position.

8 DEMOLITION

8.1 GENERAL

Special precautions are required for the demolition of prestressed concrete structures, and it is recommended that the advice of a prestressing specialist is obtained before planning the demolition.

Two references giving useful information are the FIP guide to good practice(32) and the PTI publication on the demolition of a post-tensioned slab(33).

8.2 STRUCTURES WITH BONDED TENDONS

These can normally be demolished using recognised methods of demolition. However, it is of fundamental importance that during the initial stages it is ascertained that the grouting is effective.

8.3 STRUCTURES WITH UNBONDED TENDONS

The energy introduced into the tendons in this instance is only secured by the anchorages. Release of this energy will occur over the complete length of the tendon no matter where it is cut. The sequence of releasing the tendons must be planned in detail to take into account the structure's ability to carry dead loads without prestressing and the introduction of temporary supports where necessary. Safety precautions should be taken near the anchorages although recent experimental work has shown that most of the energy is dissipated by friction, dislodging the wedges and breaking the concrete cover.

9 SPECIAL USES OF POST-TENSIONING IN BUILDING STRUCTURES

9.1 GENERAL

The following uses of post-tensioning fall outside the scope of the general design rules of this report. The basic requirements for strength and serviceability still apply and adherence to the codes with respect to crack control, deflection and stress limitation will ensure satisfactory structural performance.

9.2 TRANSFER STRUCTURES

Transfer structures are usually beams, but transfer slabs can also be effective. They are employed when concentrated loads from columns or walls do not coincide with the supporting structure and hence, the loads must be 'transferred'. Post-tensioned transfer structures are a powerful tool in reducing depth requirements and controlling deflections.

Since much of the load which is applied to transfer structures is dead load, there is advantage in balancing this with post-tensioning and hence reduce or eliminate creep deflections. The upper structure is vulnerable to relative vertical movement of its supports, so the ability to control the deflection is of great benefit.

Since the actual dead load is applied in stages the transfer structure must either be propped until the dead loads are present, or stressed in stages as the dead load accrues. The number of stages will depend on the amount of deflection control required and the sensitivity of the transfer structure to the loading and amount of post-tensioning applied. It is common when using bonded tendons in a transfer structure, to fully stress and grout individual tendons at each stage so that partially stressed tendons are not left in an unprotected state for prolonged periods.

One major concern for transfer structures is the ability to demolish them at some later time. The situation where a beam or slab is in an unstable condition, due to its prestressed state after the applied dead loads have been removed must be avoided. This may be achieved by choosing a member size that can be reinforced against the effect of the post-tensioning with a modest safety factor (say 1.05). Another alternative is to provide some means of restraining the member against the effects of the post-tensioning during demolition (e.g. using ground anchors or ballast).

If neither of the above solutions is practical, the tendons must be systematically de-stressed or demolished. This is much easier, but possibly more dangerous, with unbonded tendons, since bonded tendons will require severing at various locations to eliminate their effect on the structure.

9.3 FOUNDATION STRUCTURES

Post-tensioned slabs can also be used for the construction of foundation slabs (rafts). The balancing loads induced by the prestress can induce a more even distribution of soil pressure across the slab and this can lead to rafts significantly thinner than an un-tensioned equivalent. The aim is to produce additional downwards pressure at the mid-span between columns and corresponding upward pressure at column locations. This leads to a tendon profile which is the reverse of that normally seen in suspended slabs. At the time of raft construction there is only the dead weight of the slab to be distributed and so only limited stressing is possible. For this reason stressing is normally staged with additional prestress being applied as the weight of the building increases.

When designing post-tensioned ground beams and rafts, the effect of ground friction must be taken into account when calculating the effective pre-compression in the member (P/A). A long beam or slab may lose a considerable amount (or all) of its pre-compression in its middle zone. This may not be crucial if crack distribution steel is provided, since the vertical effect of the tendon profile is still present, and hence the benefit in spreading the column load is achieved.

The use of post-tensioning in foundations has not been exploited as much as it might be as the benefits in reduction in foundation depths, settlement control and reduction in ground pressure are worthwhile.

9.4 GROUND SLABS

The use of post-tensioning to provide joint-free ground slabs has been well applied for many years, but due to the efficient alternatives (e.g. use of steel fibres) it remains limited in its take up.

Like most ground slabs, the concrete in a post-tensioned slab relies on its tensile strength to resist the bending moments. The post-tensioning increases the effective tensile strength by means of the pre-compression and also keeps any construction joints closed. In the same way as for foundation structures described above this pre-compression may be lost in long structures by the effect of ground friction.

It is normal to keep the tendons straight (non-profiled) in ground slabs, with the tendon being as close as practicable to the concrete centroid.

The post-tensioning must be applied when the concrete is young, before early thermal and shrinkage cracking occurs. This usually means stressing within 24 hours of the concrete being cast, and special anchorages may be required to avoid bursting failure.

Both bonded or unbonded tendons can be used, although unbonded are usually the more practical for this application.

In the USA there is a large market in post-tensioning of ground slabs for house foundations, particularly those on active soils. The use in the UK has been mostly in high bay warehouses where the avoidance of joints is desirable. These have performed well (for over 20 years in some cases).

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APPENDICES

A EXAMPLES OF CALCULATIONS

B CALCULATION OF PRESTRESS LOSSES

C CALCULATION OF TENDON GEOMETRY

D CALCULATION OF SECONDARY EFFECTS USING EQUIVALENT LOADS

E CALCULATION AND DETAILING OF ANCHORAGE BURSTING REINFORCEMENT

F SIMPLIFIED SHEAR CHECK – DERIVATION OF FIGURES 19 AND 20

G VIBRATION SERVICEABILITY OF POST-TENSIONED CONCRETE FLOORS

H EFFECT OF EARLY THERMAL SHRINKAGE ON A STRUCTURAL FRAME WITH PRESTRESSED BEAMS

APPENDIX A
EXAMPLES OF CALCULATIONS

A.1 SOLID FLAT SLAB WITH UNBONDED TENDONS

A.1.1 Description, properties and loads

Equivalent frames are used to model the slab in each direction. Maximum design moments are obtained by a combination of live and dead load, with the equivalent prestress load from the tendons. Where tendon anchorages are away from the neutral axis, or are inclined to the neutral axis of the slab (either at the ends of the slab or within the span where tendons are 'stopped off'), their effect can be included in this method of analysis by the introduction of equivalent moments and/or point loads.

A floor plan of the building is shown in Figure Al together with a typical transverse sub-frame. This example analyses sub-frames on gridlines 5 and B. Calculations are carried out for full bay width.

The structure is checked both at Serviceability and Ultimate Limit States. These checks are carried out at transfer, during construction (where typically, when two weeks old, the slab may be required to carry its own weight, plus the weight of the floor above at concreting, plus associated construction loads), and under working load conditions.

Properties

Concrete:

f_{ck} = 35MPa (cylinder strength at 28 days)
f_{ci} = 25MPa (strength at transfer)
E_c = 28GPa (elastic modulus at 28 days)
E_{ci} = 21.7GPa

Bonded reinforcement:

f_y = 460MPa

Floor plan

Section A-A: Design subframe (transverse)

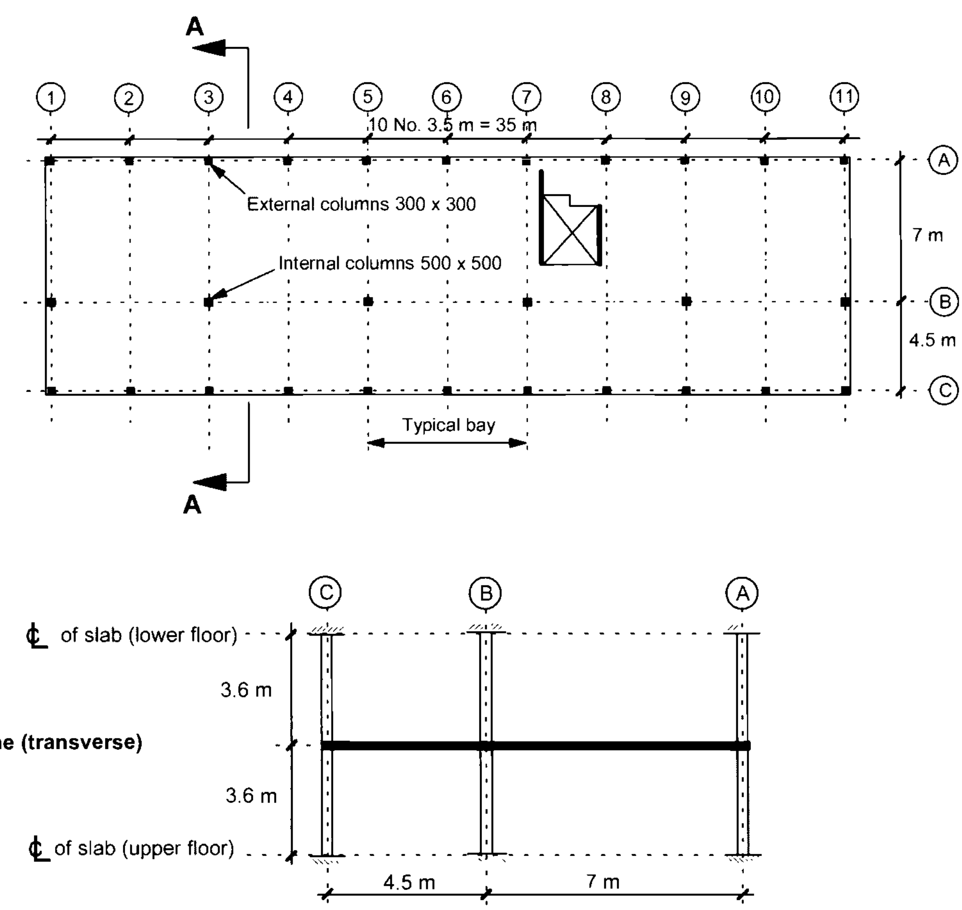


Figure A1: Floor plan and sub-frame for Example 1.

Prestressing steel:
12.9mm diameter superstrand with high-density polythene or polypropylene sheath and with lubrication/corrosion protection as detailed in Section 4.2.2.

P_k = 86kN (characteristic strength of tendon)
A_{ps} = 100mm^2 (area of tendon)
f_{pk} = P_k / A_{ps}
= 1860MPa (characteristic strength of prestressing steel)
E_{ps} = 195GPa (elastic modulus)

Loading

Imposed loading:

finishes:	partitions	1.0kN/m^2
	screed	1.2kN/m^2
	services in floor zone	0.5kN/m^2
	ceiling	0.5kN/m^2
		3.2kN/m^2
live load: typical office building		4.0kN/m^2
Total imposed loading		7.2kN/m^2

From Figure 18, for a flat slab (Section type 1) a span/depth ratio of 33 is required giving a slab depth of 210mm. However, to reduce shear reinforcement requirements (see Figure 19), a depth of 225mm is chosen.

Self-weight (using a density of 24kN/m^3)	5.4kN/m^2
Total dead load	8.6kN/m^2
Total live load	4.0kN/m^2

Check at temporary construction stage

construction load – self-weight of slab under construction above	5.4kN/m^2
additional construction load	1.5kN/m^2
	6.9kN/m^2

Share this load between two lower floors by propping

load per floor	3.45kN/m^2
self-weight of floor under design	5.40kN/m^2
Total construction load per floor	8.85kN/m^2
Worst loading = dead load + live load situation	12.6kN/m^2

Balanced load

At this stage in the calculation, it is recommended that the amount of load to be balanced is considered. The designer's experience can simplify this operation. In this example a balanced load consisting of all the dead load is chosen. (Balanced loads are discussed in more detail in Section 5.4.)

Tendon profiles

Nominal cover requirement in accordance with Eurocode 2:

for adequate cover to reinforcing and prestressing steel against corrosion XC1(15+5mm)	20mm
for 1hour fire resistance, axis distance (15+5-8)	22mm
Take nominal cover to be (see Figure A2)	25mm

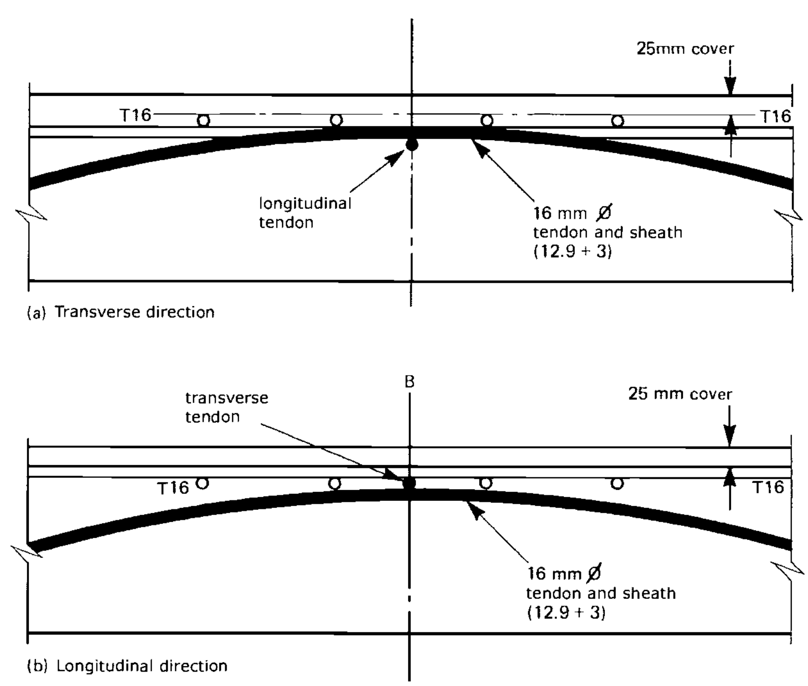


Figure A2: Tendon and reinforcing steel positioning for cover requirements.

The dimensions from the top surface of the slab to the tendons and reinforcing steel are shown in Figure A2. The positioning of the reinforcement must be considered at this stage, so as to obtain a practical arrangement of the steel at internal supports.

Preliminary shear check

Taking a slab depth of 225mm, check the punching shear capacity, and the shear capacity at the face of the column, for both internal and external columns.

Internal columns (500 × 500)
The load from the slab on to the internal columns will be greater than that due to half the span because of elastic distribution (see Section 5.6). For this example, a reasonable estimate for the increase in equivalent floor area is a factor of 1.2.

Equivalent floor area = 1.2 (3.5 + 2.25) × 7 = 48.3m^2

From Figure 19 (b) for a total imposed load of 7.2kN/m^2 and equivalent floor area of 48.3m^2, some shear reinforcement will be required for a slab of depth 225mm.

From Figure 20 the check for maximum shear stress is OK.

Edge columns (300 × 300)
Figures 19 and 20 are set up for internal columns. In order to use the figures for an edge column, the equivalent loaded area is doubled.

Equivalent floor area = 2 × 3.5 × 3.5 = 24.5m^2

From Figure 19 (a) for a total imposed load of 7.2kN/m^2 and equivalent floor area of 24.5m^2, no shear reinforcement is required for a slab of depth 225mm.

From Figure 20 the check for maximum shear stress is OK.

A.1.2 Serviceability Limit State – Transverse direction

After deciding the limiting tendon eccentricities (Figure A3) and the positions of the points of inflection (0.1 times the span, from the centre of supports) the tendon profile can be calculated; see Appendix C.

Calculation of maximum drape

Assume that the maximum drape occurs at mid-span. Using the equation of a parabola:

$$y = kx(s - x)$$

from the tendon profile calculation (Appendix C), we know that:

$$k = 2.69 \times 10^5$$

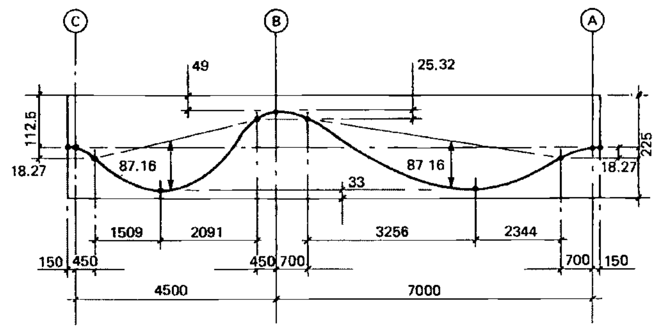


Figure A3: Transverse tendon profile.

also s = 3600mm

when x = 800mm $\quad y = 87.16$mm

At this stage losses are assumed as follows:

At transfer 10% of the jacking load

At service 20% of the jacking load.

A thorough check will be carried out after the stress calculations to check that these initial assumptions of 10% and 20% are within reason. If they are not, another estimate should be made and the procedure repeated.

Initial prestress

The initial prestress force, i.e. the jacking force, has been taken to be 70% of the characteristic strength (see National Annex to Eurocode 2 Part 1.1).

For the transverse direction, the tendons will be stressed along gridline A only.

Calculation of P_{av}

Jacking force = 0.7 × 186 = 130.20kN/tendon
Prestress force at transfer (10% losses) = 117.18kN/tendon
Prestress force at service (20% losses) = 104.16kN/tendon

Next the value of prestress force required in each span is calculated. This is done using the chosen balanced load of 8.6kN/m^2 (the dead load), the distance between points of inflection, s, and the drape, a, as shown in Figure A4.

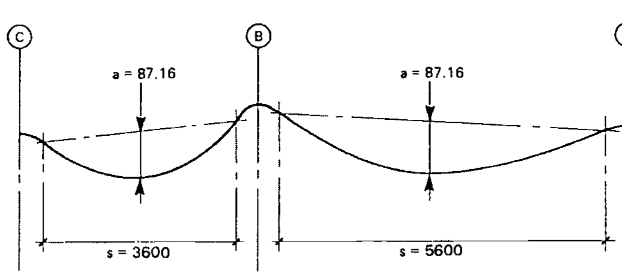


Figure A4: Drape for load balancing.

The prestress force is obtained from the following equation, which assumes a parabolic profile.

$P_{rqd} \quad = \quad ws^2 / 8a$

For span CB,

$P_{rqd} \quad = 8.6 \times 7 \times 3600^2 / (8 \times 87.16 \times 1000) = 1119\text{kN}$

Therefore number of tendons = 1119/104.16 = 10.7.
Try 11 tendons per panel.

For span BA,

$P_{rqd} \quad = 8.6 \times 7 \times 5600^2 / (8 \times 87.16 \times 1000) = 27079\text{kN}$

Therefore number of tendons = 2707/104.16 = 25.99.
Try 26 tendons per panel.

As the longer span requires more tendons than the shorter span, 15 of the tendons will be stopped off at the point of inflection in span CB, next to support B. When accurate losses are calculated, the different force profile of these shorter tendons must be taken into account.

The effect of the tendons on the slab is modelled by means of equivalent loads, as shown below. Equivalent loads are discussed in more detail in Section 5.4 and Appendix D. It should be noted that the portions of the cable from the edges of the slab to gridlines A and C are horizontal and so do not contribute to the equivalent loads.

The equivalent load, w, between any two points of inflection for the chosen number of tendons is given by:

$w \quad = \quad 8anP_{av} / s^2$

where

n = number of tendons
a = drape at the point considered
s = as shown in Figure A.4
P_{av} = average force provided by each tendon.

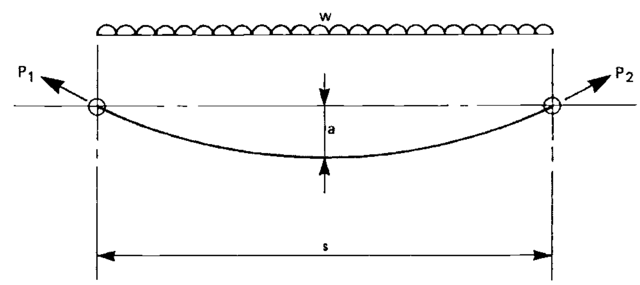


Figure A5: Calculation of equivalent loads due to tendon forces.

Table A1: Calculations of equivalent loads due to transverse tendons, at transfer and after all losses.

Equivalent loads at transfer						
Full length tendons (n = 11)	**C**	**Span**	**B**	**B**	**Span**	**A**
$n \times P_{av}$ (kN)	1289.0	1289.0	1289.0	1289.0	1289.0	1289.0
a (mm)	18.3	–87.2	25.3	25.3	–87.2	18.3
s (mm)	900	3600	900	1400	5600	1400
w (kN/m)	233.0	–69.4	322.1	133.1	–28.7	96.3
Short length tendons (n = 15)						
$n \times P_{av}$ (kN)	–	–	1757.7	1757.7	1757.7	1757.7
a (mm)	–	–	25.3	25.3	–87.2	18.3
s (mm)	–	–	900	1400	5600	1400
w (kN/m)	0	0	439.2	181.5	–39.1	131.3
Total w (kN/m)	233.0	–69.4	761.3	314.6	–67.8	227.6

Equivalent loads after all losses						
Full length tendons (n = 11)	**C**	**Span**	**B**	**B**	**Span**	**A**
$n \times P_{av}$ (kN)	1145.8	1145.8	1145.8	1145.8	1145.8	1145.8
a (mm)	18.3	–87.2	25.3	25.3	–87.2	18.3
s (mm)	900	3600	900	1400	5600	1400
w (kN/m)	207.1	–61.7	286.3	118.3	–25.5	85.6
Short length tendons (n = 15)						
$n \times P_{av}$ (kN)	–	–	1562.4	1562.4	1562.4	1562.4
a (mm)	–	–	25.3	25.3	–87.2	18.3
s (mm)	–	–	900	1400	5600	1400
w (kN/m)	0	0	390.4	161.3	–34.8	116.7
Total w (kN/m)	207.1	-61.7	676.7	279.7	-60.2	202.3

When tendons are anchored within the span, as in this example, additional equivalent loads may be generated by the end condition. These must be included in the frame analysis when obtaining the bending moments and shear force diagrams. The forces consist of a vertical and horizontal component of the tendon force applied at the anchor.

Figure A6 shows the effect of an anchorage in terms of additional equivalent loads on the slab.

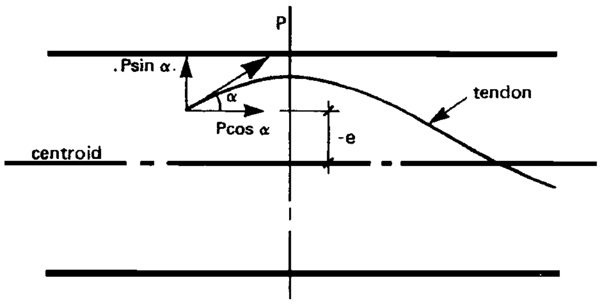


Figure A6: Equivalent loads at anchorages.

The vertical component of the tendon force is easily calculated, and should be applied to the slab as a vertical point load at the point where the tendon is anchored. The horizontal component forms a positive moment about the centroid, owing to its eccentricity from the centroid of the section, and should be applied in this form to the slab.

It should be noted that the position of the tendon at the anchorage can be arranged so that the tendon is both horizontal (no vertical force) and at the centroid of the section (no eccentric moment). In this example the anchorages at the ends of the full-length tendons fulfil this requirement and no additional loads are generated.

Vertical force $= P \sin \alpha$

Eccentric moment at the point of inflection $= P \cos a \times e$

For a parabolic tendon $dy/dx = 2ax/s^2$

and so $\sin \alpha = 2a/s$

Therefore $\sin \alpha = 2 \times 25.32 = 0.1125$

$\cos \alpha = 0.9937$

Eccentricity of tendon, e, = −112.5 + 25(cover) + 16(diameter of un-tensioned reinforcement) + 8(half the tendon diameter) + 25.32(drape)

= −38.18mm

At transfer:

P = 1757.70kN

$P \sin \alpha$ = 197.82kN

$Pe \cos \alpha$ = −66.68kNm

After all losses:

P = 1562.4kN

$P \sin \alpha$ = 175.84kN

$Pe \cos \alpha$ = −59.276kNm

The equivalent loads from the tendons, the anchors and the superimposed loads are then used to calculate design moments and shears by any convenient method of structural analysis. This is normally done using an appropriate computer program.

At Serviceability Limit State, an elastic method of analysis should be used for analysing post-tensioned flat slabs, and patterned loading should be used in multi-span situations.

Summary of prestress equivalent loads

Table A2: Summary of uniformly distributed equivalent loads from transverse tendons.

	C	Span	B	B	Span	A
Equivalent loads at transfer (kN/m)	233.0	−69.4	761.3	314.6	−67.8	227.6
Equivalent loads after all losses (kN/m)	207.1	−61.7	676.7	279.7	−60.2	202.3

Table A3: Summary of additional equivalent loads due to internal anchorages.

	Vertical force	Eccentric moment
At transfer	197.8kN	66.7kNm
After all losses	175.8kN	59.3kNm

Check that prestress loads total to zero

Upward loads
 = $(61.7 \times 3.6) + (60.2 \times 5.6) + 175.8$
 = 735.8kN

Downward loads
 = $(207.1 + 676.7) \times 0.45 + (279.7 + 202.3) \times 0.7$
 = 735.11kN

The small difference between these values is due to earlier approximations. The equivalent loads were altered to total zero at this point to enable consistent calculation of secondary moments.

Calculation of stresses

$$f_t = P/A_c - Pe/z_t + M_A/z_t + M_s/z_t$$

$$f_b = P/A_c + Pe/z_b - M_A/z_b - M_s/z_b$$

$$A_c = 7 \times 0.225 \times 10^6 = 1.575 \times 10^6 \text{mm}^2$$

As the section being considered is rectangular and symmetrical about the centroid, z_t and z_b are equal.

$$z_t = z_b \equiv z = bh^2 / 6 = 5.91 \times 10^7 \text{mm}^3$$

As this example is a flat slab, analysed by the equivalent frame method, the allowable stresses are as detailed in Table 4 (Section 5.8.1). To increase ease of construction, un-tensioned reinforcement has been deliberately omitted from spans by keeping the tensile stresses below $0.3f_{ctm,fl}$ (transfer – where f_{ck} is replaced by the initial concrete strength) and $0.3f_{ctm,fl}$ (service).

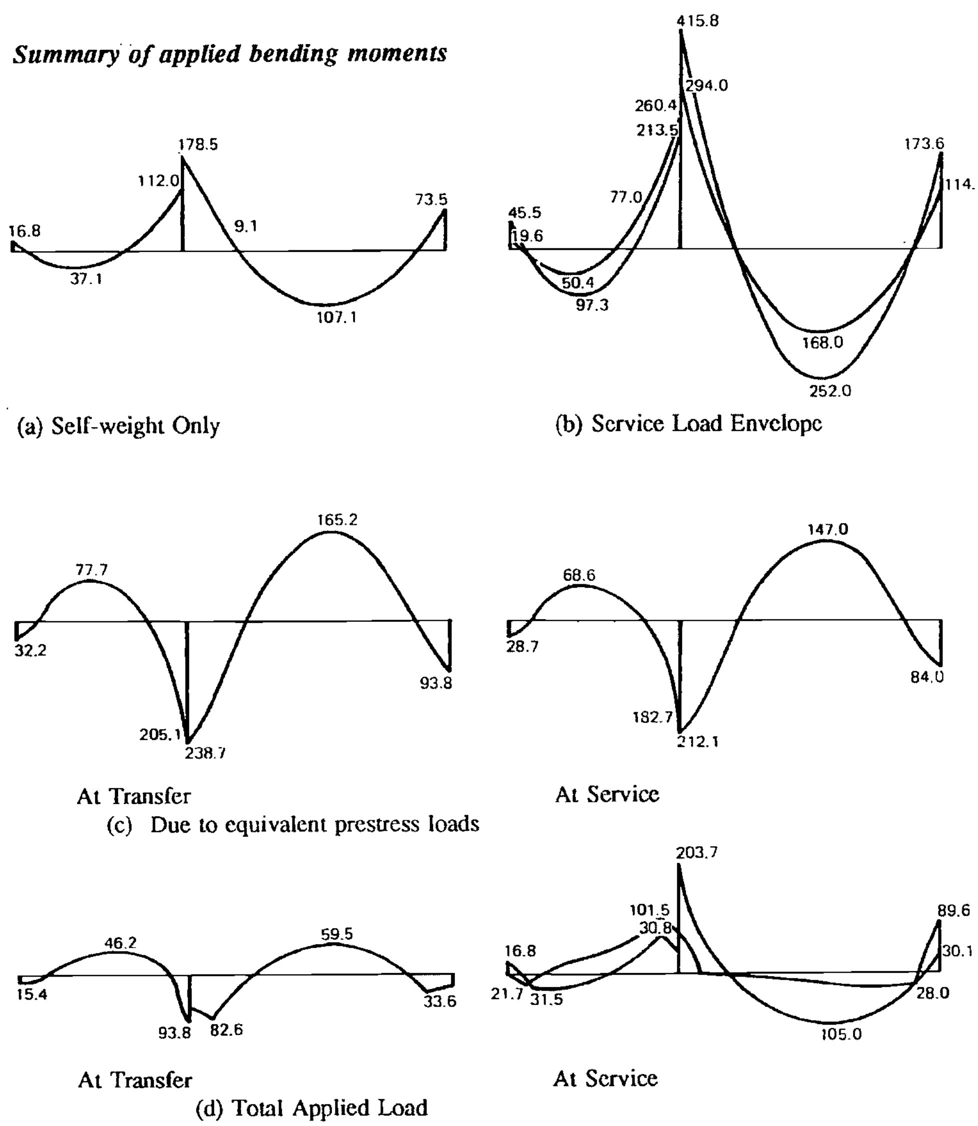


Figure A7: Applied bending moment diagrams.

Table A4: Stresses at transfer for the transverse direction.

Zone		Stress due to prestress* (MPa)	Stress due to self-weight (MPa)	Total stress (MPa)	Allowable stress (MPa)
C	**top**	1.364	–0.280	1.084	7.5
	bottom	0.272	0.280	0.552	–2.65
CB (hogging)	**top**	–0.492	0.534	0.043	–1.06
	bottom	2.128	–0.534	1.594	10
B	**top**	5.406	–1.891	3.515	7.5
	bottom	–1.538	1.891	0.355	–2.65
B	**top**	4.755	–1.429	3.326	7.5
	bottom	–0.887	1.429	0.542	–2.65
BA (sagging)	**top**	2.509	–0.154	2.355	10
	bottom	1.359	0.154	1.513	–1.06
BA (hogging)	**top**	–0.862	1.790	0.928	–1.06
	bottom	4.730	–1.790	2.940	10
A	**top**	2.504	–0.006	2.498	7.5
	bottom	1.364	0.006	1.370	–2.65

* These values include prestress secondary effects

Table A5: Stresses after all losses for the transverse direction.

Zone		Stress due to prestress* (MPa)	Stress due to self-weight (MPa)	Total stress (MPa)	Allowable stress (MPa)
C	**top**	1.27	–0.77	0.50	–3.31
	bottom	0.18	0.77	0.95	10.5
CB (sagging)	**top**	–0.23	1.49	1.26	14
	bottom	1.68	–1.49	0.19	–1.32
CB (hogging)	**top**	0.83	–1.31	–0.48	–1.32
	bottom	0.62	1.31	1.93	14
B	**top**	2.67	–2.67	0.00	–3.31
	bottom	0.77	2.67	3.44	10.5
B	**top**	5.31	–7.04	–1.73	-3.31
	bottom	–1.88	7.04	5.16	10.5
BA (sagging)	**top**	–0.77	4.27	3.50	14
	bottom	4.21	–4.27	–0.06	–1.32
A	**top**	3.13	–2.93	0.20	–3.32
	bottom	0.30	2.93	2.23	10.5

Hogging and sagging values are given where they both occur in one zone. Each span is split into three zones, from the end to $2L/10$, from $2L/10$ to $8L/10$ and from $8L/10$ to L.

In this example, the construction load is smaller than the load at service and larger than that at transfer. This means that the construction case is not likely to be a governing situation and so the stresses are not calculated.

A.1.3 Loss calculations

At this stage the losses should be calculated accurately to check that the initial assumptions of 10% at transfer and 20% at service were reasonable. The method for calculating the various steps of loss is given in Appendix B.

Full-length tendons

Short-term losses

a) Losses due to friction

$P_x = \quad P \times e^{-\mu x(\alpha' + \varpi)}$

Table B1 gives recommended values for the coefficients μ and ϖ:

$\mu = 0.06$ and $\varpi = 0.05$rads/m

Deviated angle per metre, $\alpha' = (16 \times \text{total drape})/L^2$

Total drape $= (18.27 + 25.32)/2 + 87.16 = 108.96$mm
(the same for both spans)

Span CB
$\alpha' = (16 \times 108.96 \times 10^{-3})/4.5^2 = 0.086$rad/m

Span BA
$\alpha' = (16 \times 108.96 \times 10^{-3})/7^2 = 0.036$rad/m

Jacking force $= 130.2$kN

Forces after friction losses (see Figure A8) are:

$P_A = 130.2$kN

$P_B = 130.2 \times e^{-7 \times 0.06(0.036+0.05)} = 125.6$kN

$P_c = 125.58 \times e^{-4.5 \times 0.6(0.086+0.05)} = 121.1$kN

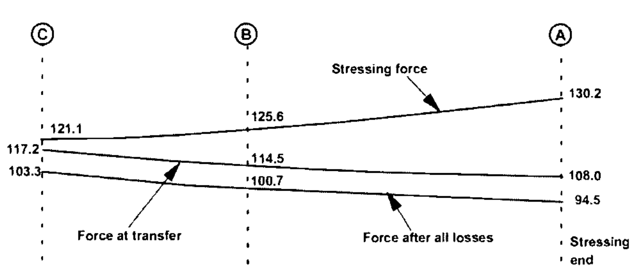


Figure A8: Force profiles for full-length tendons.

b) Losses due to wedge set

Force loss at anchorage, $\delta P_w = 2p'l'$

where
p' = slope of force profile
l' = length of tendon effected by draw-in
$= \sqrt{[(\Delta \times E_{ps} \times A_{ps})p']}$

Take wedge draw-in,
$\Delta = 6$mm
$E_{ps} = 195$kN/mm^2
$A_{ps} = 100$mm^2
$p' = (p_A - p_C)/(L_1 + L_2)$
$= (130.2 - 121.07) / (7 + 4.5)$

Hence
$l' = \sqrt{[(6 \times 10^{-3} \times 195 \times 100)/0.79]}$
$= 12.14$m

As l' is greater than the length of the tendon, at stressing anchorage:

$\delta P_w = (\Delta \times E_{ps} \times A_{ps})/l + (p' \times l)$
$= (6 \times 195 \times 100)/(11.5 \times 10^3) + 0.79 \times 11.5 = 19.30$kN

and at dead end:

$\delta P_w = (\Delta \times E_{ps} \times A_{ps})/l - (p' \times l)$
$= (6 \times 195 \times 100)/(11.5 \times 10^3) - 0.79 \times 11.5 = 1.04$kN

Forces after friction losses and wedge set (see Figure A8):

$P_A = 130.2 - 19.30 = 110.9$kN
$P_B = 125.58 - [(19.30 - 1.04) \times 4.5/11.5] - 1.04$
$= 117.4$kN
$P_C = 121.07 - 1.04 = 120.0$kN

c) Losses due to early thermal shrinkage

$\delta P_{etsh} = \varepsilon_{etsh} \times E_{ps} \times A_{ps}$

where
$\varepsilon_{etsh} = 100 \times 10^{-6}$ (see Section 3.3)

$\delta P_{etsh} = 100 \times 10^{-6} \times 195 \times 100 = 1.95$kN

d) Elastic losses

$\delta P_{es} = \varepsilon_{es} \times E_{ps} \times A_{ps}$

where
$\varepsilon_{es} = 0.5 \times f_{co} / E_{ci}$

f_{co} is the stress in the concrete adjacent to the tendon. Since this is unlikely to be critical, the stress is calculated at a representative point and will be taken as uniform over the whole tendon length.

f_{co} = 1.984MPa
ε_{es} = $0.5 \times 1.984/(21.7 \times 10^3)$ = 4.57×10^{-5}
δP_{es} = $4.57 \times 10^{-5} \times 195 \times 100$ = 0.89kN

Prestress at transfer
Prestress force at A = 110.9 – 1.95 – 0.89 = 108.0kN
Prestress force at B = 117.4 – 1.95 – 0.89 = 114.5kN
Prestress force at C = 120.0 – 1.95 – 0.89 = 117.2kN

Long-term losses

a) Relaxation of steel

δP_r = 1000-hour relaxation value × relaxation factor × prestress force at transfer

From Table B2 in Appendix B values are taken for an initial jacking force equal to 70% of the characteristic strength.

loss due to relaxation = 2.5%
relaxation factor = 1.5

Therefore, $\delta P_r = 2.5\% \times 1.5 \times P_x = 3.75\% \times P_x$

δP_{rA} = 0.0375×110.01 = 4.13kN
δP_{rB} = 0.0375×116.50 = 4.37kN
δP_{rC} = 0.0375×119.14 = 4.47kN

b) Shrinkage of concrete

δP_{sh} = $\varepsilon_{sh} \times E_{ps} \times A_{ps}$
ε_{sh} = 300×10^{-6} (see Section 3.3)
δP_{sh} = $300 \times 10^{-6} \times 195 \times 100$ = 5.85kN

c) Creep of concrete

δP_{cr} = $\varepsilon_{cr} \times E_{ps} \times A_{ps}$
ε_{cr} = $f_{co}/E_{ci} \times \phi$

where
ϕ = creep coefficient (see Appendix B) = 2.0

f_{co} = 1.984MPa
ε_{cr} = $(1.984 \times 2) / (21.7 \times 10^3)$ = 1.83×10^{-4}
δP_{cr} = $1.83 \times 10^{-4} \times 195 \times 100$ = 3.57kN

Prestress after all losses
Prestress force at A = 108.0 – 4.13 – 5.85 – 3.57 = 94.5kN
Prestress force at B = 114.5 – 4.37 – 5.85 – 3.57 = 100.7kN
Prestress force at C = 117.2 – 4.47 – 5.85 – 3.57 = 103.3kN

Short tendons

Friction, early thermal shrinkage and elastic losses are the same as for the full-length tendons, as are the long-term losses. The effect of wedge set is different as the tendon length is different and must be recalculated.

Force at dead end = $125.58e^{-0.45 \times 0.06(0.086 + 0.05)} = 125.14$kN

Losses due to wedge set
δP_w at dead end = $(\Delta \times E_{ps} \times A_{ps})/l - (p' \text{x } l)$
= $6 \times 195 \times 100/7450 - (130.2 - 125.14)$ = 10.64kN

δP_w at stressing anchorage = $(\Delta \times E_{ps} \times A_{ps})/l + (p' \text{x } l)$
= $6 \times 195 \times 100/7450 + (130.2 - 125.14)$ = 20.77kN

Forces after friction losses and wedge set:
P_A = 130.2 – 20.77 = 109.4kN
P_B = 25.58 – 45(20.77 – 1064)/745 – 10.64 = 114.3kN

Prestress at transfer
Prestress force at A = 109.4 – 0.89 – 1.95 = 106.6kN
Prestress force at B = 114.3 – 0.89 –1.95 = 111.5kN

Prestress after all losses
Prestress force at A = 106.6 – 4.07 – 5.85 –3.57 = 92.5kN
Prestress force at B = 111.5 – 4.25 – 5.85 –3.57 = 97.8kN

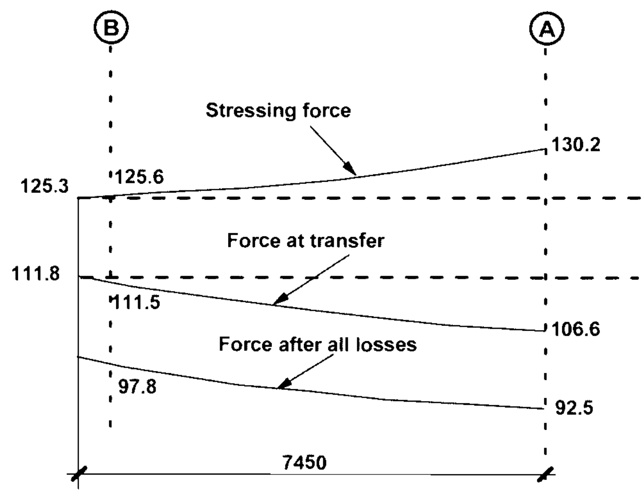


Figure A9: Force profiles for short tendons.

Check of the assumed losses against the actual looses

At transfer
Average short-term loss for span CB (full length tendons only)

= [(130.2 – 117.2)/130.2 + (130.2 – 114.5)/130.2]/2 × 100
= 1.0%

Average short-term loss for span BA (both tendon lengths)

= 100[(11/26)(130.2 – 114.5 + 130.2 0150 108.0) + (15/26)(130.2 – 111.5 + 130.2 – 106.6)]/(130.2 × 2)
= 15.5%

Average overall short term loss = (11 + 15.5)/2 = 13.2%

After all losses
Average long-term loss for span CB (full length tendons only)

= [(130.2 – 103.3)/130.2 + (130.2 – 100.7)/130.2]/2 × 100
= 21.6%

Average long-term loss for span BA (both tendon lengths)

= 100[(11/26)(130.2 – 100.7 + 130.2 – 94.5) + (15/26)(130.2 – 97.8 + 130.2 – 92.5)]/(130.2 × 2)
= 26.6%

Average overall long term loss = (21.6 + 26.6)/2 = 24.1%

Although the assumed losses of 10% and 20%, respectively, have been exceeded recalculation is not considered necessary, as it will not cause an increase in the number of tendons. Also the calculation of stresses for the correct losses are unlikely to exceed the allowable values.

A.2 FINITE ELEMENT DESIGN EXAMPLE

A.2.1 Description, properties and loads

In order to demonstrate the procedures and methods detailed in Section 5.7.3 the following simple example is provided. For the purpose of the example a typical structural grid of 8.5m × 8.5m with a slab thickness of 250mm is used, supported on square column of 400mm × 400mm in size (see Figure A10).

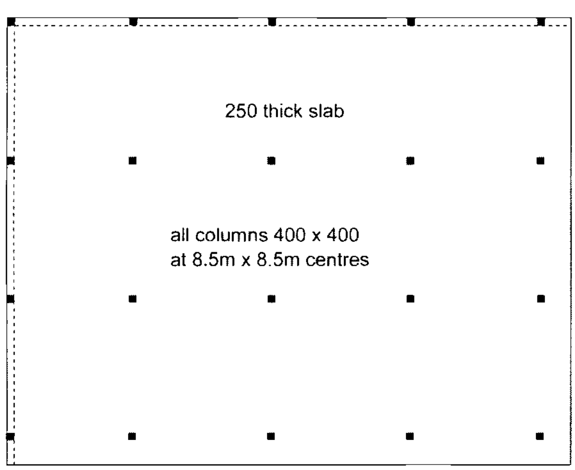


Figure A10: Slab arrangement.

Design criteria

Slab thickness	= 250mm
Column size	= 400mm × 400mm
Column length	= 3600mm above
	= 3200mm below
Concrete grade	= C35/45 (35MPa cylinder strength /45MPa cube strength at 28 days).
Initial concrete grade	= Cube strength at time of stressing 25MPa (equivalent cylinder strength is 20MPa)

The following design loadings will be used:

Self weight	– Calculated on the basis of normal weight concrete.
Imposed dead	– Allow 1.8kN/m^2 (to include services, finishes, partitions etc).
Imposed live	– Allow 5.0kN/m^2 (based on required use).
Edge loading	– Allow 7.5kN/m run (Dead) for cladding etc.

It is assumed that the structure is for use as an office.

Other design criteria

For the purpose of this example the following details have been chosen:

Cover to bonded reinforcement	=	30mm
Cover to prestressing ducts	=	40mm
Size of top reinforcement	=	H20
Size of bottom reinforcement	=	H12

For the purposes of this example a 'bonded' system has been chosen. The system details for use in this example are:

System	= 5 strand flat oval ducts
Duct size	= 20mm × 70mmm flat oval
Strand size	= 12.9mm super strand
Strand area	= 100mm^2
Strand G.U.T.S	= 186kN

The losses in the prestressing system due to friction, elastic shortening, creep shortening, shrinkage, wobble, wedge set and strand relaxation would normally be calculated for each tendon used in the design based on its length, profile, jacking force etc as detailed in Appendix B. However for the purposes of this example it is assumed that the force in each strand, after all losses, is 100kN (the calculation of the prestress losses is usually included as part of the software analysis package).

Load combinations
The load combinations used for a post-tensioned concrete design are similar to those for a reinforced concrete design. Load cases are included for the serviceability limit state (SLS) and the ultimate limit state (ULS). It is also necessary to check the stresses in the slab at the time of initial prestressing ('at transfer' load case). For this the prestress loads are applied, before the long-term losses have occurred, and usually before any imposed dead or live loads are in place. The load case represents the situation just after the prestressing tendons are stressed. At this time it is unlikely that the full imposed dead or live loads are present but it may be possible that a construction load is applied.

The 'at transfer' load case is important because the prestress generally applies upward forces to the structure, to resist the applied gravity loads. When the applied loads are not present it is possible to induce bending moments and actions in the opposite sense to those that are generally expected. In certain situations this can result in the need to reinforce in the top of mid-span zones, for example. This is particularly the case where transfer beams are present.

The load factors used for each load case should be determined from reference to EN 1990: 2002 (E). For this example we will use the following load cases. The various loadings to be applied to the structure are summarised below.

G_{k1} = Self-weight of structure
G_{k2} = Imposed dead loading
Q_{k1} = Imposed live loading
Q_{k2} = Imposed dead loading at transfer (the 'construction load')
P_{BL} = Prestress equivalent ('balance') loads (Loading loading induced by prestressing tendons). **Note:** this loading includes any secondary load effects.
P_{SL} = Prestress secondary loads. **Note:** This is the secondary loading induced by the application of the prestressing tendons. It is generally only applied for ultimate limit states and should not, of course, be applied simultaneously with the Equivalent Loads.

The following combinations are obtained from BS EN 1990: 2002 (E) (Eurocode 0)[24], Clause 6.5.3 and Table A1.1:

Serviceability Limit State
(for Category B: office areas $\psi_0 = 0.7$, $\psi_1 = 0.5$, $\psi_2 = 0.3$)

Characteristic Load Combination
(from Expression (6.14 b) of EC0)

$$(G_{k1} + G_{k2}) + P_{BL} + Q_{k1}$$

Frequent Load Combination
(from Expression (6.15 b) of EC0)

$$(G_{k1} + G_{k2}) + P_{BL} + 0.5Q_{k1}$$

Quasi-Permanent Load Combination
(from Expression (6.16 b) of EC0)

$$(G_{k1} + G_{k2}) + P_{BL} + 0.3Q_{k1}$$

For this example the majority of the variable loads (Q_k) is live and therefore the *Frequent load combination* will be used for checking the section stresses etc at SLS. For plant rooms etc, where the imposed loading is likely to be more permanent in nature, the characteristic combination should be used for the stress check at the SLS.

The *Quasi-Permanent Load combination* is used for checking the long-term deflections.

As discussed previously it is also necessary to include an additional load case to represent the 'at transfer' condition. This will be taken as:

$$G_{k1} + a\, P_{BL} + 0.5\, (Q_{k2})$$

where
a = ratio of the prestress force before and after the long term losses have occurred.

Ultimate Limit State
The load combination for the ultimate limit state will be selected with reference to Eurocode 0, Clause 6.4 and Annex A1.3 (Table A1.2 (B)).

The ultimate local combination is taken (from Expression 6.10 of Eurocode 0) as:

$$1.35(G_{k1} + G_{k2}) + 0.9P_{SL} + 1.5Q_{k1}$$

For the purposes of this example external lateral loads have not been included within any of the load combinations. If these were required to be included, for example if the slab formed part of the lateral load carrying system, then the applied lateral loading would need to be incorporated with the necessary combinations as required by the relevant codes.

It should be noted that the internal lateral forces (the axial forces from the prestressing tendons, for example) are included in the analysis of the slab as part of the prestress loads.

Note: Most software packages will include the elastic shortening effects of such loads and the more sophisticated packages can incorporate the creep shortening, concrete shrinkage and thermal effects. The designer should be aware of these effects and their implications on the structure and should take due account depending on the size, construction sequence, release details of the slab (see Section 3.3).

A.2.2 Analysis

The analysis of the slab can be undertaken with either a plane frame or finite element analysis package. The choice between these is often based on the complexity of the slab layout. For a more complex layout the finite element packages generally offer a more convenient method of analysis.

A number of proprietary software packages are available to undertake such analysis. For the purposes of this example a finite element analysis will be undertaken.

The slab is analysed as a whole floor including any dropped areas of slab, downstand beams etc. The analysis also generally includes the columns and walls above and below the slab. The designer can generally select the support conditions at the remote ends of the walls and columns. The selection of the finite element size and the arrangement of elements can have a significant effect on the accuracy of the results from the finite element analysis. Generally a minimum of eight elements should be provided for each span, although specific advice should be sought from the software supplier.

For this example the element layout shown in Figure A11 will be used.

The perspective view of the floor plate is shown Figure A12.

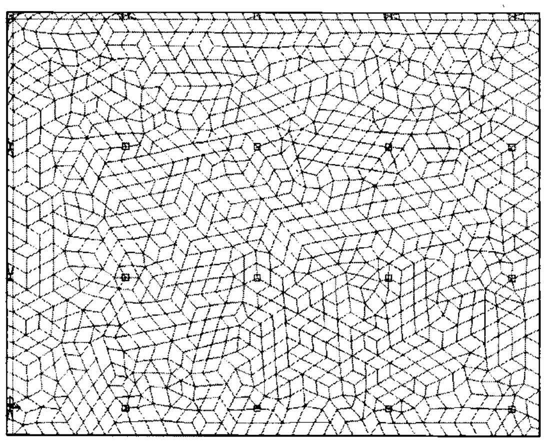


Figure A11: Finite element mesh for example.

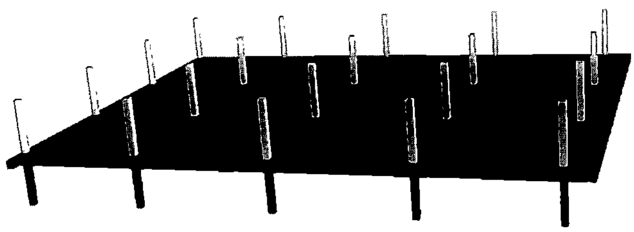


Figure A12: Perspective view of slab system.

Tendon layout

The arrangement of the prestressing tendons within the slab is obviously a result of the design process. For a plane frame analysis it is possible to automate the iterative process. This is not as straightforward for a finite element as any number of areas in each direction could potentially be critical sections. In order to progress towards the optimum solution the designer should first make an initial selection.

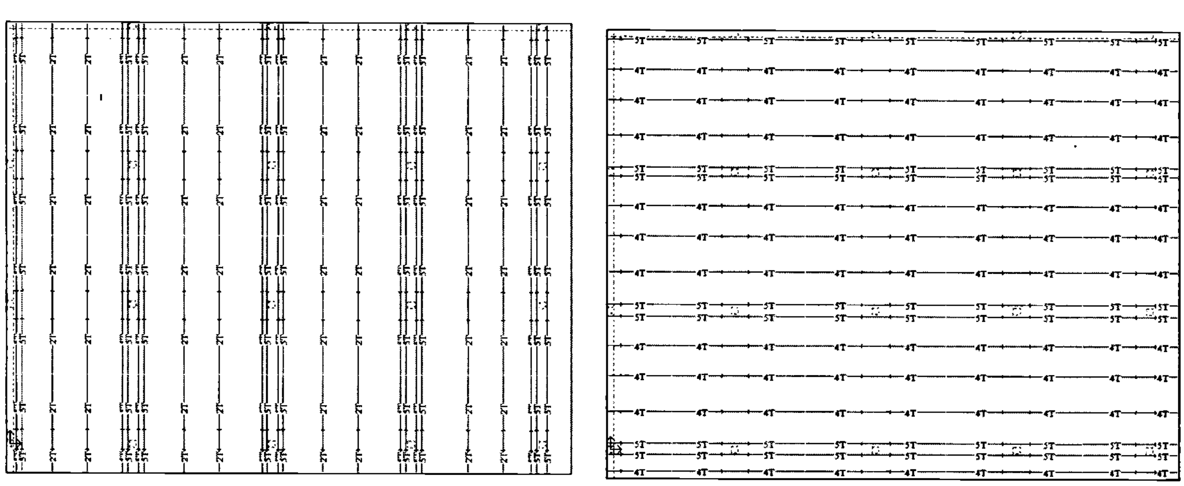
For most designs it will be acceptable to select the number of tendons to give an average axial compression of around 1MPa. The tendons will generally be placed with low points in the mid-span zones and high points over the columns. The tendons will generally be set to neutral axis position at all edges. The high and low points of the tendons are set to suit the cover and layering requirements. The tendons are profiled to give smooth parabolic profiles. Such profiles are well suited to slabs loaded with uniformly distributed loads. Different profiles may be required where the slab is loaded by point loads or other non-uniform arrangements.

The layout of the tendons can also be selected by the designer. A number of arrangements are possible as discussed in Chapter 2. For the purpose of this example a banded and distributed layout is used as shown in Figure A13.

For a finite element analysis the tendons are actually input in their actual position with their equivalent loads applied to the slab as line loads, point loads and bending moments, as appropriate. The initial layout of the tendons will be amended throughout the design process by adding or removing tendons or strands, changing the their profiles and/or moving their position, as necessary to obtain compliance with the relevant checks.

'Design strips'

To interpret the result of the finite element analysis it is necessary to identify 'design strips' (defined in Section 5.7.3). The lines of 'zero shear' are located using the ULS load combination although the design is normally carried using the SLS load combinations. The lines of 'zero shear' for this example are shown in Figure A14. Engineering judgement may be required to position these lines.



a) Banded tendons in y-direction **b) Distributed tendons in x-directions**

A13: Tendon layout.

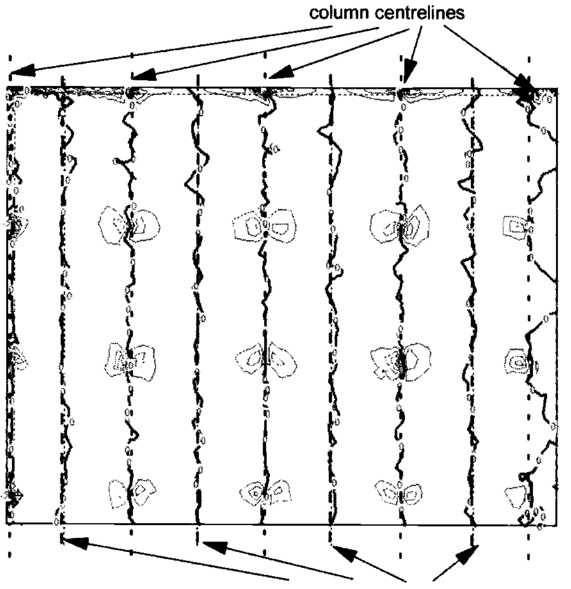


Figure A14: Lines of zero shear.

For designs where the column layout is not regular and/or the loading is not uniform it is possible that the line of zero shear will not always run parallel to the column centre lines. It will however generally be acceptable to adopt a straight line for any particular span of the structure to define the 'design sections'.

The 'design strips' along a typical line of columns are shown in Figure A15 (see Section 5.7.3).

For this example the layout all of the 'design strips' for the whole slab in each direction are shown in Figure A16.

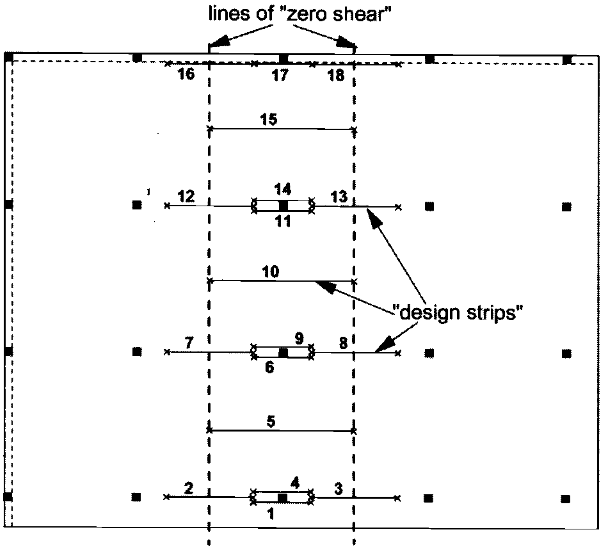


Figure A15: 'Design strips' for a typical line of columns.

A.2.3 Results from analysis

With most finite element software packages it is possible to present the results of the analysis in a number of ways including contour plots, perspective views, tabulated text etc,

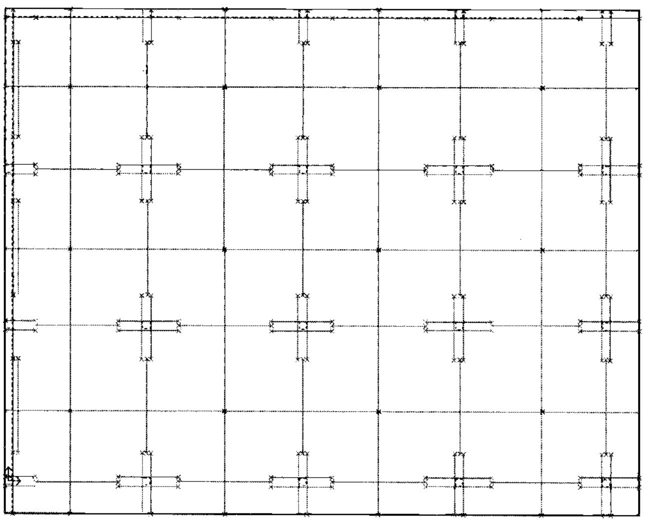


Figure A16: Full set of 'design strips' for example.

and the sheer amount of data produced can be overwhelming at first. For the purposes of this example the design checks required on the 'design strips' shown in Figure A15 are investigated in detail. Each of the 'design strips' is uniquely numbered and the analysis results for each limit state are summarised in Table A6.

In these tables tensile stresses are shown as positive. The values given are the extreme fibre stresses in the top and bottom of the slab based on the elastic analysis and using the full cross-sectional properties of the concrete. No allowance has been made, at this stage, for cracking etc. The values quoted under the heading 'Centroid' are the stresses at the mid-depth of the slab.

The stresses at each 'design strip' should be compared with the allowable stresses given in Section 5.8.1, Table 5.

The allowable stresses for each load case are:

Frequent load combination

Tensile stress (with bonded reinforcement)
= $1.2f_{ctm}$
= 3.84MPa
(f_{ctm} = 3.2MPa Table 3.1 Eurocode 2)

Tensile stress (without bonded reinforcement)
= $0.4f_{citm}$
= 1.28MPa
(f_{ctm} = 3.2MPa Table 3.1 Eurocode 2)

Compressive stress
= $0.4f_{ck}$
= 14.0MPa
(f_{ck} = 35MPa (cylinder strength))

Table A6: Concrete stresses at Serviceability Limit State.

Frequent combination					**'At transfer' condition**			
'Design strips' No	**Concrete stresses (MPa)**				**'Design strips' No**	**Concrete stresses (MPa)**		
	Top	**Bottom**	**Centroid**			**Top**	**Bottom**	**Centroid**
1	−2.87	−1.32	−2.09		1	−4.83	−0.108	−2.47
2	−1.31	−0.11	−0.709		2	−1.84	0.172	−0.836
3	−1.33	−0.194	−0.764		3	−1.85	0.049	−0.9
4	0.369	−4.03	−1.83		4	−3.4	−0.909	−2.16
5	−3.41	0.951	−1.23		5	−1.63	−1.27	−1.45
6	3.6	−6.07	−1.24		6	−1.35	−1.56	−1.46
7	0.227	−2.65	−1.21		7	−1.52	−1.34	−1.43
8	0.222	−2.56	−1.17		8	−1.54	−1.21	−1.38
9	3.66	−6.08	−1.21		9	−1.05	−1.8	−1.43
10	−2.1	−0.341	−1.22		10	−0.856	−2.01	−1.43
11	3.42	−5.72	−1.15		11	−1.08	−1.63	−1.36
12	0.328	−2.77	−1.22		12	−1.32	−1.54	−1.43
13	0.387	−2.73	−1.17		13	−1.23	−1.52	−1.37
14	4.53	−6.93	−1.2		14	−0.582	−2.24	−1.41
15	−3.65	1.29	−1.18		15	−1.88	−0.888	−1.39
16	−0.189	−0.93	−0.559		16	−0.503	−0.697	−0.6
17	0.82	−5.06	−2.12		17	−2.02	−3.14	−2.58
18	−0.259	−0.95	−0.605		18	−0.588	−0.718	−0.653

'At transfer' condition

Tensile stress (without bonded reinforcement)
= $0.3f_{citm}$
= 0.66MPa
(f_{citm} = 2.2MPa Table 3.1 Eurocode 2)

Compressive stress
= $0.4f_{ci}$
= 8.0MPa
(f_{ci} = 20MPa (cylinder strength))

Care should be taken to ensure that the correct values for the allowable stresses are used where sections are classed either 'with bonded reinforcement' or 'without bonded reinforcement'. For this design example a bonded prestressing system has been used. This allows the use of the 'with bonded reinforcement' value for the majority of locations. It is possible however, even when using a bonded prestressing system, to have 'design strips' where tendons are not passing through. This can occur for example where 'design strips' are located between tendon bands. In such situations the 'with bonded reinforcement' allowable stress values should not be used unless a mat of ordinary reinforcement is provided in the tensile face. The ordinary reinforcement, in such situations, would need to have sufficient cross-sectional area to carry the full moment over the width of the design section and would be suitably anchored into a zone of lower applied stress.

Serviceability stress checks (Compression)

For the design example it can be seen, by reference to the above tables, that the allowable compressive stresses are not exceeded at any of the 'design strips'.

The maximum value for the frequent load case is −6.93MPa (compression) in the bottom of 'design strip' No. 14. This is considerably less than the allowable value of −14.0MPa. For the 'at transfer' load case the maximum compressive stress is −4.83MPa in the top of 'design strip' No. 1. This is less than the allowable value of −8.0MPa.

Serviceability stress checks (Tensile)

As discussed above we must make an assessment at this point as to whether to use the 'with bonded reinforcement' or 'without bonded reinforcement' values. By referring to Figure A15 we can establish the following:

- 'Design strip' numbers 1, 4, 6, 9, 11, 14 and 17 all occur at column locations where we will always provide a mat of bonded reinforcement (minimum reinforcement) and we know that the bonded tendons will be positioned over the column and therefore through the 'design strip'. We can therefore use the 'with bonded reinforcement value' for the top stresses for these 'design strips'.
- For 'design strip' numbers 16 and 18 we would normally provide an edge cage of reinforcement in the top and bottom of the slab which will allow us to use the 'with bonded reinforcement value' for the top and bottom stresses for these 'design strips'.
- Similarly we know that the bonded tendons will be positioned to pass through the 'design strip' for numbers 5, 10 and 15. We can therefore use the 'with bonded reinforcement value' for the bottom stresses for these 'design strips'.
- The remaining 'design strips' (numbers 2, 3, 7, 8, 12 and 13) potentially fall between the tendon bands and therefore for our initial checks we should use the 'without bonded reinforcement' values for the allowable stresses.

It is noted that if the stresses at these sections were found to exceed the allowable values the designer would have the option of providing a suitable mat of bonded reinforcement and rechecking against the higher stress limit.

For the 'at transfer' load case it is assumed that the allowable tensile stress is the 'without bonded reinforcement' value. This is a sensible first starting point as frequently the stresses for this load case are in the opposite sense to the other load cases and therefore generally the tendons and any bonded reinforcement are not present in the tensile face. Again the designer would have the option of providing a suitable mat of bonded reinforcement and rechecking against the higher stress limit at specific locations if the lower allowable stresses were exceeded.

The tensile stresses experienced by the slab can be compared with the values given in Table 5. The comparison is presented in Tables A7. Here only the positive (tensile) stresses have been shown and are compared, for each design section, with the allowable values.

It can be seen that the allowable stresses are not exceeded for the 'at transfer' condition and the serviceability stress checks are therefore satisfied for this load case.

For the frequent load case however, the applied tensile stresses are exceeded at one location ('design strip' number 14), as highlighted in bold. It is noted that this strip is located adjacent to the internal columns where the highest stresses in the top of the slab would be expected. At all other locations the applied stresses are within the allowable values.

At this point in the design process the designer has a number of choices. The design parameters (i.e. the concrete grade, span lengths, slab thickness etc) could be varied or the prestress levels adjusted by adding more tendons or changing the tendon profiles. In this way the design could be adjusted such that the allowable concrete stresses are not exceeded.

Alternatively the designer may choose to undertake a more rigorous assessment of the slab and consider the predicted crack widths in accordance with Section 5.8.3. In order to demonstrate this procedure the example will proceed on this basis.

Crack width assessment

For the purposes of this example 'design strip' No. 14 is checked.

From Table A1 the stresses at Section 14 are:

Top of slab = 4.53MPa (tension)
Bottom of slab = –6.93MPa (compression)

In addition to the stresses the corresponding forces applied across this 'design strip' can be obtained from the analysis as shown in Table A8.

Table A8: Data from analysis for 'design strip' No. 14.

Corresponding forces applied across 'design strip' No. 14					
Axial (kN)	Lateral (kN)	Vertical (kN)	Bending (kNm)	Strip width (m)	Strip thickness (mm)
–992	–54	327	–198	3.31	250

Table A7: Tensile stresses as Serviceability Limit State compared with limiting values.

Frequent combination						'At transfer' condition				
'Design strip' No	Tensile stresses (MPa)					'Design strip' No	Tensile stresses (MPa)			
	Applied		Allowable				Applied		Allowable	
	Top	Bottom	Top	Bottom			Top	Bottom	Top	Bottom
1			3.84	1.28		1			0.66	0.66
2			1.28	1.28		2		0.172	0.66	0.66
3			1.28	1.28		3		0.049	0.66	0.66
4	0.369		3.84	1.28		4			0.66	0.66
5		0.951	1.28	3.84		5			0.66	0.66
6	3.6		3.84	1.28		6			0.66	0.66
7	0.227		1.28	1.28		7			0.66	0.66
8	0.222		1.28	1.28		8			0.66	0.66
9	3.66		3.84	1.28		9			0.66	0.66
10			1.28	3.84		10			0.66	0.66
11	3.42		3.84	1.28		11			0.66	0.66
12	0.328		1.28	1.28		12			0.66	0.66
13	0.387		1.28	1.28		13			0.66	0.66
14	**4.53**		3.84	1.28		14			0.66	0.66
15		1.29	1.28	3.84		15			0.66	0.66
16			3.84	3.84		16			0.66	0.66
17	0.82		3.84	1.28		17			0.66	0.66
18			3.84	3.84		18			0.66	0.66

Using the above information an equivalent cracked section in accordance with Section 5.8.3 can be analysed. For the cracked section any tensile stresses in the concrete are ignored (see Figure 36).

The following values are known:

d_S = 210mm (the effective depth of the ordinary bonded reinforcement)
d_{PS} = 196mm (the effective depth of the prestressing tendons)
h = 250mm (the depth of the section).

In addition, from the tendon layout plan (see Figure A13), 20 No. prestressing strands cross design section 14. Since this design strip is at a column location the requirement for minimum reinforcement (see Section 5.8.8) is $0.00075A_c$. The value of A_c relates to the full width between lines of zero stress (see Figure 33), which for the column adjacent to 'design strip' No. 14, is:

A_c = 8275 × 250 × 0.00075 = 1551mm^2

For this example a mat of 10 H20 bars at 200 centres is provided over the column. This gives a reinforcement area of 3140mm^2, which exceeds the minimum requirement. Hence:

A_{ps} = 20 × 100 = 2000mm^2
A_s = 3140mm^2

The unknown values F_S and F_{PS} can be expressed in terms of x and f_{cc} by using the strain compatibility diagram (see Figure 40c).

The long-term value of the elastic modulus for concrete is:

$E_{c,long} = E_{c,short}/(1+\phi)$ = 11.33GPa, where ϕ is taken as 2.

For this example the solution gives x = 112.5mm and f_{cc} = 8.9MPa

Using these values the stress in the reinforcement and prestressing tendons f_S and f_{PS} is calculated to be 136.1MPa and 117.2MPa, respectively. It should be noted that the stress in the tendons from prestressing is treated as a load and is not included on the resistance side.

Hence F_S = 136.1 × 3140/1000 = 427.4kN

and F_{PS} = 117.2 × 2000/1000 = 234.4kN (flexure)

The section stress details are shown in Figure A17.

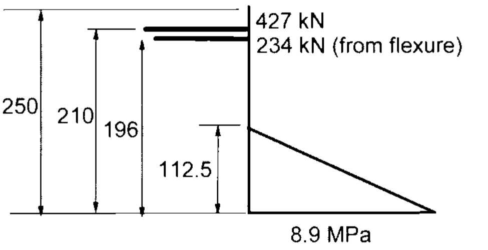


Figure A17: Stress distribution in section of 'design strip' No. 14.

Once the stress in the bonded reinforcement has been established the corresponding predicted crack width can be calculated in accordance with Eurocode 2, Clause 7.3.4. For this example the crack width is calculated to be 0.09mm, compared with the allowable crack width of 0.3mm (See UK National Annex to Eurocode 2, Table 7.1N).

If the allowable crack width exceeds the permitted value the designer should either revise the design parameters (slab depth, prestress levels etc) or add additional bonded reinforcement and recalculate the cracked section, steel stresses and the resultant crack width, until compliance is achieved.

Similar crack width checks should be undertaken for all other 'design strips' where the allowable stresses are exceeded.

Ultimate Limit State

All of the forces applied to the 'design strip' can be obtained from the analysis (see Table A9). These are then processed using the design formulae given in Eurocode 2 to establish the requirements for Ultimate Limit State. Depending on the location it may be necessary to provide additional bonded reinforcement to supplement the capacity of the prestressing tendons.

Table A9: 'Design strip' forces at Ultimate Limit State.

'Design strip' No.	Axial (kN)	Lateral (kN)	Vertical (kN)	Torsion (kN-m)	Bending (kN-m)
1	0.191	0.971	–210	–1.29	–150
2	3.22	9.3	–0.677	119	–43.2
3	0.222	4.17	30	–98.6	–38.7
4	2.34	3.97	517	25.5	–379
5	12.8	4.55	3.58	–34.4	635
6	–0.145	–2.4	–535	–38.4	–550
7	–0.7	–13.9	–105	–141	–258
8	1.59	0.923	–138	1.24	–259
9	4.33	–0.518	528	115	–525
10	0.279	–3.15	13	–30.7	438
11	11	–0.956	–603	–127	–511
12	–17.5	7.57	39.1	23.8	–254
13	–11.2	–1.86	–44	–189	–250
14	3.26	0.72	679	34.6	–592
15	1.22	–5.54	59.9	–46.6	659
16	–188	–74.3	–159	–16.9	–38.8
17	179	41	–317	–20.1	–243
18	–190	–28.7	–156	–12.2	–36.7

For the Ultimate Limit State it is necessary to undertake design checks for the bending and shear (including punching shear) capacities.

A.2.4 Reinforcement areas

The ordinary bonded reinforcement required at each location in the slab is based on the areas calculated from the following:

- the ultimate limit state analysis
- the serviceability stress checks
- the reinforcement provided to satisfy the crack width calculation
- the reinforcement (if any) added to enhance the punching shear resistance at column locations
- the minimum reinforcement requirements (see Section 5.8.8).

For the serviceability stress checks it is noted that bonded reinforcement is required at all locations where the stress exceeds $0.4f_{ctm}$ (Table 5). For this example a bonded prestressing system has been used which can be assumed to provide the necessary bonded reinforcement in most locations. It is possible however that design strips are located between tendon bands and that additional ordinary bonded reinforcement is required. For this example the value $0.4f_{ctm}$ equates to a stress of $0.4 \times 3.2 = 1.28$MPa. Table A7 shows that this stress is exceeded at 'design strip' number 14. This design strip occurs at a column where the bonded tendons are present.

The minimum reinforcement for a flat slab is given in Section 5.8.8. At each column location there is a requirement to provide at least 0.075% of the gross concrete cross-section. For the purposes of this check the gross concrete cross-section is based on the full bay width (i.e. between the lines of zero shear).

For this example the bay width is typically 8.5m. Therefore the minimum reinforcement area at the supports should be $0.00075 \times 8500 \times 250 = 1594\text{mm}^2$. This reinforcement is placed at maximum of 300mm centres over the zone 1.5 times the slab depth either side of the width of the column (i.e. a width of $2 \times 1.5 \times 250 + 400 = 1150$mm). A minimum of six H20 bars at 225mm centres is chosen. This ensures that there are 6 No. bars in the required zone thus giving a total area of 1884mm^2.

From the crack width checks a mat of 10 H20 bars at 200mm centres over the supports (this applies at the columns adjacent to design sections 6, 9, 10 and 14) has already been chosen. At all other locations six H20 bars at 225mm centres are provided in order to satisfy the minimum reinforcement provision.

At all slab edges H10 U-bars at 200mm centres laced with at least two H10 longitudinal bars top and bottom is provided, as required by Section 5.8.8. Reinforcement is also provided in the triangular unstressed area between anchorages as detailed in Section 5.12.

A.2.5 Deflection checks

For most slabs it will be acceptable to base the assessment of the slab deflections on the elastic deflections, factored as detailed in Section 5.8.4 to make a notional allowance for the effects of creep. The deflection assessment should be made after the modification in the E value, if appropriate, to allow for the effects of cracking, if applicable.

For the design example it has already been established that the allowable stresses are exceeded at two of the columns where stresses have been checked. At these locations it has been established that the resultant crack widths are acceptable. It is therefore only necessary to modify the E value of the slab elements local to the effected columns. The elements selected for the modification of the E value are indicated by shading in Figure A18.

The above plan shows the element mesh overlaid with 'design strips'. The allowable tensile stresses are exceeded at two column locations and at these points a cracked section analysis has been required. Although not specifically checked in this example, it is assumed that similar stresses will be experienced on the other similar column lines. This gives a total of six column locations where the E value is required to be modified.

The E value of the concrete is revised for the elements shown highlighted, namely those within the length of the affected design strips. It is noted that the elements selected appear somewhat irregular, however this is due to the original arrangement of the element mesh. The designer could choose to refine the element mesh to give a closer fit to the required areas, however it will generally be acceptable and conservative to modify an area slightly larger than that required to suit the length of the design strips.

The value of the reduction in the E value will vary depending on the extent of the cracking and should be determined based on the specific project. For this example we will choose to apply a reduction of 30%.

The modification to the E value will have the effect of redistributing the support bending moments into the slab and increasing the slab deflections. Only a single iteration of the modification of the E value is considered necessary for most typical structures.

Following the modification of the E value and with the load factors given in Table 6 of Section 5.8.4 the maximum slab deflection, for the design example, is 12.1mm. This is compared to the allowable deflection of span/500. For this example the allowable deflection is $8500/500 = 17$mm, therefore the actual deflection is within the allowable limits.

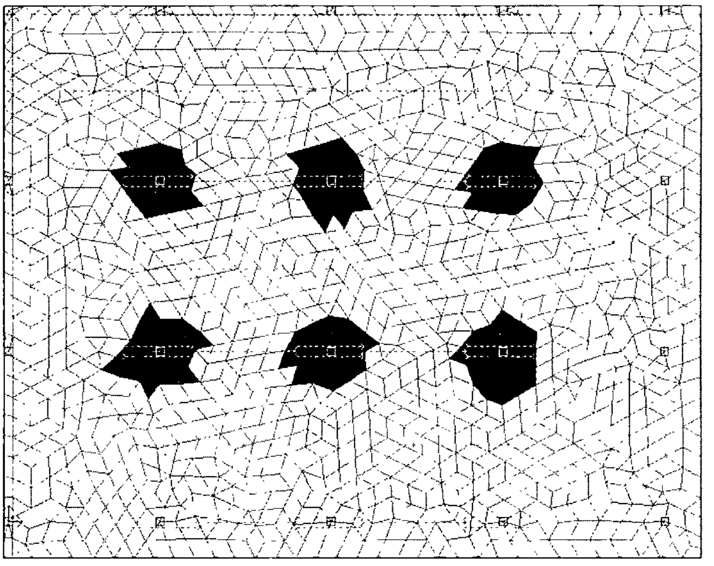


Figure A18: Modification of E value.

A.3 PUNCHING SHEAR DESIGN FOR EXAMPLE A1

A.3.1 Properties

The properties and dimensions are as follows:

f_{ck}	=	35MPa
column width	=	500mm
slab depth	=	225mm
d transverse direction	=	184mm
d longitudinal direction	=	168mm
d_{Av}	=	176mm

First control perimeter (at $2d$), $u_1 = (4 \times 500) + (4\pi \times 176)$ = 4212mm

A.3.2 Applied shear

The reduction in the shear at an internal column from the vertical reaction of the prestressing tendons is calculated as follows:

Transverse direction

Width to consider for reduction (slab depth + column width, from Section 5.9.3)	=	725mm
Number of tendons passing through this width	=	3
Prestress force/tendon after all losses	=	101kN
Average value of $s = (5600 + 3600)/2$	=	4600mm
Value of a	=	87.2mm
Shear reduction from prestressing tendons = $(8 \times 87.2 \times 3 \times 101)/4600$	=	46.0kN

Longitudinal direction

Width to consider for reduction of shear	=	725mm
Number of tendons passing through this width	=	2
Prestress force/tendon after all losses	=	101kN
Average value of s	=	5600mm
Value of a	=	81.5mm
Shear reduction from prestressing tendons = $8 \times 81.5 \times 3 \times 101/5600$	=	23.5kN
Total shear reduction from prestressing tendons, V_{Pd} (kN) = 45.5 + 23.5	=	69kN

Applied shear from analysis, V_{Ed}	=	898kN
$V_{Ed} - \gamma_p V_{Pd}$ (kN) = 898 – 69	=	829kN
Applied moment in the transverse direction	=	152kNm

From Expressions 6.39 and 6.41 of Eurocode 2:

$\beta = 1 + 0.6 \times (152/829) \times 1000[4 \times 500 + 4\pi \times 184]/[500^2/2 + 500^2$
$+ (4 \times 500 \times 184) + (16 \times 184^2) + (2\pi \times 500 \times 184)] = 1.255$

Hence $V_{eff} = 1.255 \times 829 = 1040$kN

A.3.3 Shear resistance

Check first control perimeter (2d from column).

$v_{Rd,c} = C_{Rd,c}\, k(100\rho f_{ck})^{1/3} + 0.1\sigma_{cp}$ (Eurocode 2, Expression 6.47)

where

$C_{Rd,c}$	= 0.18/1.5	= 0.12
k	= MIN{1+√(200/d); 2}	= 2
ρ		= 0.58%

Side 1 (Side on short span in transverse direction)

Prestressing force/tendon	=	100.7kN
Number of tendons	=	11
Width of slab	=	7000mm
$\gamma_p P/A$ (after all losses) = $0.9 \times 100.7 \times 11 \times 1000/(7000 \times 225)$	=	0.63MPa
$0.1\gamma_p P/A$	=	0.063
$v_{Rd,c}$ (1) = $0.12 \times 2 \times (0.58 \times 35)^{1/3} + 0.063$	=	0.72MPa
$V_{P/A}$(1) = $0.063 \times (500 + \pi \times 184) \times 184/1000$	=	12.5kN
$V_{Rd,c}$ (1) = $0.72 \times (500 + \pi \times 184) \times 184/1000$	=	142.4kN

Side 2 (Side on long span in transverse direction)

Number of long tendons	=	11
Prestressing force/tendon	=	100.7kN
Number of short tendons	=	15
Prestressing force/tendon	=	97.8kN
Width of slab	=	7000mm
$\gamma_p P/A$ (after all losses) = $0.9 \times (100.7 \times 11 + 97.8 \times 15) \times 1000/(7000 \times 225)$	=	1.47MPa
$0.1\gamma_p P/A$	=	0.15
$v_{Rd,c}$ (1) = $0.12 \times 2 \times (0.58 \times 35)^{1/3} + 0.15$	=	0.80MPa
$V_{P/A}$(1) = $0.15 \times (500 + \pi \times 184) \times 184/1000$	=	29.18kN
$V_{Rd,c}$ (1) = $0.8 \times (500 + \pi \times 184) \times 184/1000$	=	156.53kN

Side 3 & 4 (Side on short span in longitudinal direction)

Prestressing force/tendon	=	104.7kN
Number of tendons	=	22
Width of slab	=	6720mm

$\gamma_p P/A$ (after all losses) = $0.9 \times 104.7 \times 22 \times 1000/(6720 \times 225)$	=	1.37MPa
$0.1\gamma_p P/A$	=	0.14
$v_{Rd,c}$ (1) = $0.12 \times 2 \times (0.58 \times 35)^{1/3} + 0.14$	=	0.79MPa
$V_{P/A}(1) = 0.14 \times (500 + \pi \times 168) \times 168/1000$	=	23.7kN
$V_{Rd,c}$ (1) = $0.71 \times (500 + \pi \times 168) \times 168/1000$	=	136.7kN
Total $V_{P/A} = 12.5 + 29.18 + 2 \times 23.7$	=	89.1kN
Total $V_{Rd,c} = 142.4 + 159 + (2 \times 136.7)$	=	575kN
$v_{Rd,c}$(Av)	=	0.78MPa

$V_{eff} > V_{Rd,c}$ Thus shear reinforcement is required.

A.3.4 Shear reinforcement

Check outer perimeter

Effective shear, $V_{eff,outer} = V_{eff} - V_{P/A}$	=	950.6kN
$v_{Rd,c} = C_{Rd,c}\, k(100\rho f_{ck})^{1/3}$	=	0.65MPa
$u_{out,ef} = V_{eff,outer} / (v_{Rd,c} \times d_{av})$	=	8249mm
Distance from column = $(8249 - 4 \times 500)/(2\pi)$	=	995mm
Distance of outer link perimeter from column = $995 - (1.5 \times 176)$	=	731mm
$f_{ywd,ef}$	=	296MPa
A_{sw}/s_r (from Expression 6.52 of Eurocode 2) = $(1.282 - 0.75 \times 0.65) \times 4212/(1.5 \times 296)$	=	7.56
$s_r = 0.75d$	=	132mm
A_{sw}/perimeter = 7.56×132	=	998mm^2
Maximum distance of links around perimeter = 1.5×176	=	264mm
Number of perimeters required = $(731/176 - 0.5) / 0.75 + 1$	=	6

The required number of links on the various perimeters is given in Table A10.

Table A10: Required number of links.

Perimeter	**Distance from column**	**Number of links required for different link diameters**		
		8 (mm)	**10 (mm)**	**12 (mm)**
1	$0.5d$	20	13	9
2	$1.25d$	20	13	12
3	$2d$	20	15	15
4	$2.75d$	20	18	18
5	$3.5d$	21	21	21
6	$4.25d$	24	24	24
Total number of links required		125	104	99

APPENDIX B
CALCULATION OF PRESTRESS LOSSES

B.1 FRICTION LOSSES IN THE TENDON

Friction losses can be calculated in accordance with BS 8110[5]. However, it is the view of the Working Party that the following calculations are more realistic. The losses are due to the friction resulting from the change in angle of the tendon and unintentional 'wobble' in the tendon. Both effects are considered in the common formula for friction:

$$P_x = P_0 e^{-\mu(\alpha + \varpi x)} \qquad \text{(B1)}$$

where

P_x = force at distance x from stressed end
P_0 = stressing force (at anchor)
μ = friction coefficient
α = angle change in tendon from anchor to point considered (radians)
ϖ = 'wobble' factor (radians/m)

This is equivalent to Equations 58 and 59 in BS 8110, Part 1, Clause 4.9.

The value of the friction coefficient will depend on the prestressing system chosen and, in the case of bonded systems, the state of the strand in terms of rust film. In the absence of detailed information on friction coefficient and wobble factor from the prestress system supplier, it is recommended that the factors in Table B1 are used.

Table B1: Typical friction coefficients and wobble factors.

	Unbonded tendons	**Bonded tendons**
Friction coefficient, μ	0.06	0.20
Wobble factor (radians/m), ϖ	0.05	0.0085

For slab type structures with unbonded tendons it is normally reasonable to assume a uniform angle change per unit length. This angle change can be obtained by calculating the total angle turned through over the full length of the tendon and dividing by the full tendon length. Alternatively a simple method based on the typical drape and span can be used. Figure B1 illustrates the geometry of a typical parabolic tendon with a reverse parabola at the support. The tangent to the curve at the point of inflection extends through points 'c' and 'a'.

Thus the slope

θ_a = $\tan^{-1}[(2 \times \text{total drape})/A]$

Similarly, using points 'b' and 'c', slope θ_b can be obtained. Over the span L the total deviated angle = $\theta_a + \theta_b$.

The average deviated angle per unit length, α', is therefore:

α' = $2(\theta_a + \theta_b)/L$

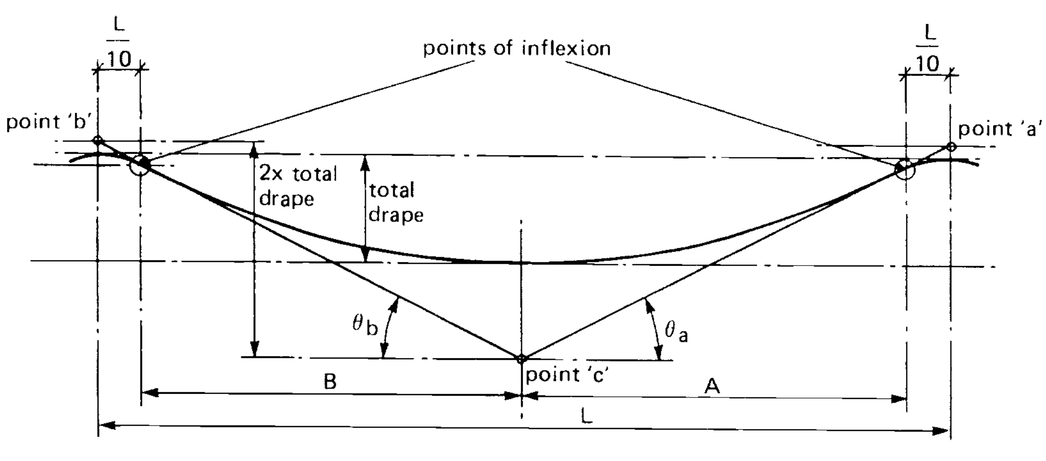


Figure B1: Typical geometry of tendon profile for internal span.

On the assumption that the point 'c' is in the centre of the span, this may be simplified to:

$\alpha' = \quad (16 \times \text{total drape})/L^2$

In such cases, Equation B1 may be rewritten as:

$$P_x = P_0 e^{-\mu x(\alpha' + \omega)} \qquad \text{(B2)}$$

The prestress force profile after friction losses can now be drawn.

B.2 WEDGE SET OR DRAW-IN

Most post-tensioning systems used in buildings depend on a wedge-based system for anchoring. In order for the wedges to grip, there must be a small movement of the strand into the anchorages. This inward movement reduces the prestress and the amount of movement depends on the particular prestressing system employed; a typical value is 6mm. The draw-in effect is as shown in Figure B2.

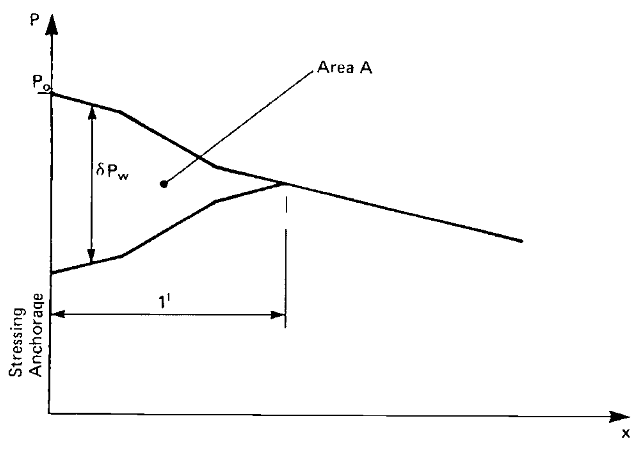


Figure B2: Loss of prestress due to wedge draw-in.

The force loss is calculated as follows:

$$\text{Area A} = \Delta \times E_{ps} \times A_{ps}$$

$$= \int_0^{l'} \delta P_w \, dx \qquad \text{(B3)}$$

where
Δ = wedge draw-in
δP_w = force loss
E_{ps} = modulus of elasticity of tendon
A_{ps} = area of tendon
l' = length of tendon affected by draw-in

If it can be assumed that the tendon has a uniform angle change per unit length, then the force profile is approximately linear. Consequently, if l' is less than the length of the tendon, then:

$$l' = \sqrt{[(\Delta \times E_{ps} \times A_{ps})/p']}$$

where
p' = slope of the force profile

and
$\delta P_w = 2 \times p' \times l'$ at anchorage

The force loss, within the length l', is then given by:

$$\delta P_w = 2\,p'(l' - x)$$

If the wedge draw-in affects the whole length of the tendon, then:

$\delta P_w = (\Delta \times E_{ps} \times A_{ps})/l + (p' \times l)$ at stressing anchorage

$\delta P_w = (\Delta \times E_{ps} \times A_{ps})/l - (p' \times l)$ at dead end anchorage.

B.3 ELASTIC SHORTENING OF THE STRUCTURE

As strands are tensioned, the structure will shorten elastically. In most building floors, this shortening is insignificant in terms of losses, but may be significant in highly stressed beams. The force loss is given by:

$$\delta P_{es} = \varepsilon_{es} \times E_{ps} \times A_{ps}$$

where
ε_{es} = $0.5 \times (f_{co} / E_{ci})$
f_{co} = stress in the concrete adjacent to the tendon after transfer
E_{ci} = modulus of elasticity of the concrete at time of transfer.

In the formula for ε_{es} above, the factor of 0.5 takes account of the averaging effect of several tendons stressed sequentially (BS 8110, Part 1, Clause 4.8.3). If this is not the case, this factor may have to be modified.

B.4 SHRINKAGE OF THE CONCRETE

BS 8110 covers this subject extensively in Section 4.8.4 of Part 1 and Section 7.4 of Part 2. Special care should be taken in thin members (e.g. slabs) subjected to low humidity (such as in some buildings) when shrinkages of more than 400×10^{-6} can occur.

The force loss is given by:

$$\delta P_{sh} = \varepsilon_{sh} \times E_{ps} \times A_{ps}$$

where
ε_{sh} = shrinkage strain of concrete.

B.5 CREEP OF CONCRETE

Creep loss is based on the stress in the concrete at the level of the tendons. These losses are extensively covered by BS 8110, Part 1, Section 4.8.5 and Part 2, Section 7.3. They can have a very large effect in highly stressed thinner members.

The force loss is given by:

$$\delta P_{cr} = \varepsilon_{cc} \times E_{ps} \times A_{ps}$$

where
ε_{cc} = $(f_{co} \times \phi)/E_{ct}$
ϕ = creep coefficient (BS 8110, Part 2, Figure 7.1).

For ribbed structures, an effective thickness should be obtained from the ratio of volume to surface area.

B.6 RELAXATION OF THE TENDONS

The stress in the tendons reduces with time because of the relaxation of the steel. The amount of relaxation depends on the type of strand and the initial stress. Figure B3 illustrates typical relaxation curves for various types of strand and load levels.

The force loss is given by:

δP_r = 1000-hour relaxation value × relaxation factor × the prestress force at transfer.

The 1000-hour relaxation value is given in the draft BS EN 10138[16], for Class 2 low relaxation steel as referred to in Eurocode 2. Data for the relaxation of this type of steel are given in Table B2.

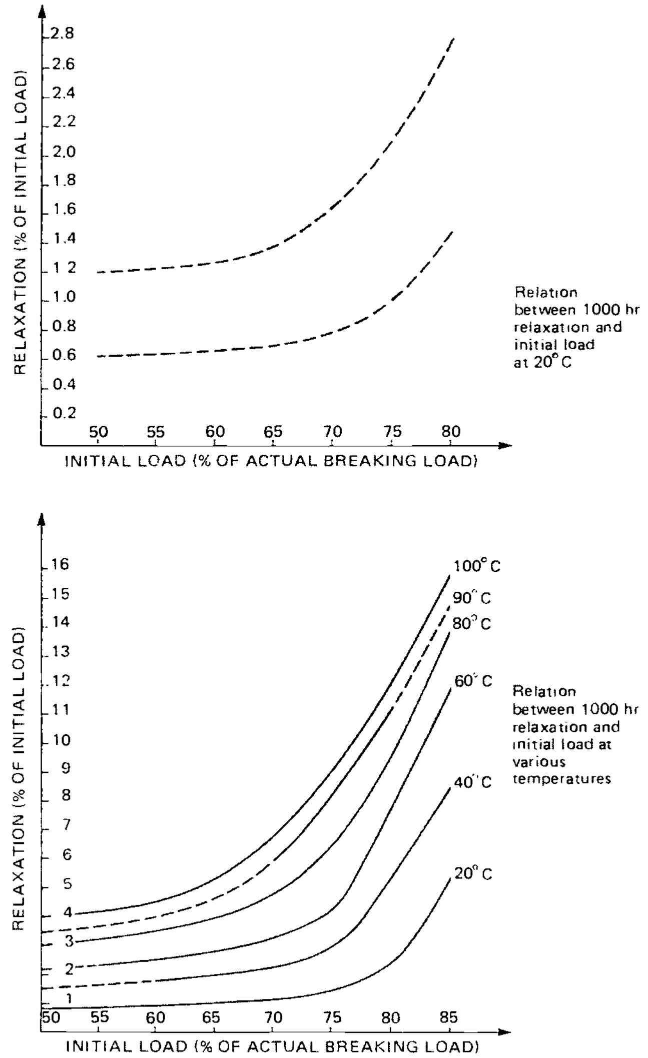


Figure B3: Relaxation curves for different types of strand at various load levels.

Table B2: Relaxation for Class 2 low-relaxation steel.

Force at transfer as a % of characteristic strength of tendon	1000-hour relaxation	Relaxation factor	Force loss as a % of force at transfer
80%	4.5%	1.5	6.75%
70%	2.5%	1.5	3.75%
60%	1.0%	1.5	1.50%

Notes:

1. Characteristic strength of tendon $= f_{pu} \times A_{ps}$
2. The 1000-hour relaxation values can be replaced with the manufacturer's values if available.

APPENDIX C
CALCULATION OF TENDON GEOMETRY

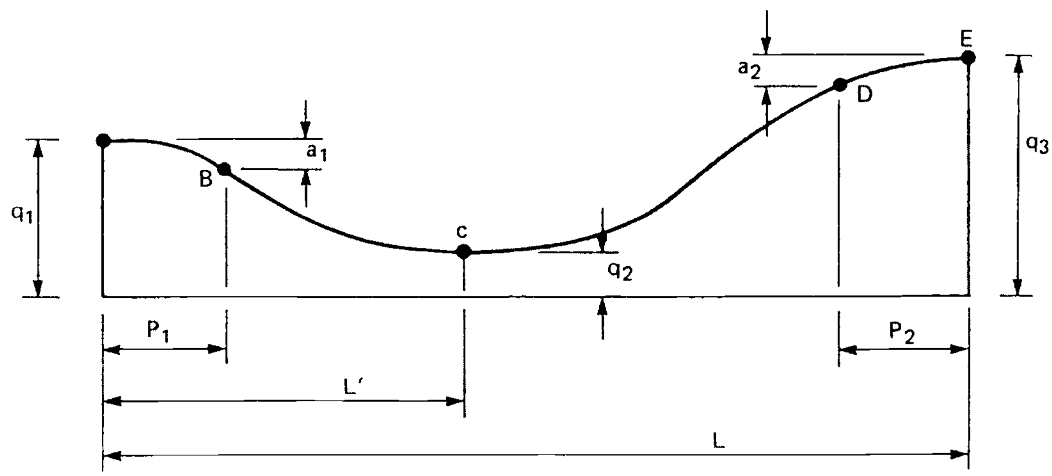


Figure C1: Tendon geometry.

Consider the three parabolas AB, BCD and DE.

Parabola	AB	y	=	$k_1 x^2$
	BCD	y	=	$k x^2$
	DE	y	=	$k_2 x^2$

For parabola AB: $-a_1 = k_1 p_1^{\,2}$

Similarly for DE, $-a_2 = k_2(-p_2)^2$

Let $Q_1 = q_1 - q_2$ and $Q_2 = q_3 - q_2$

Then for parabola BCD:

$$(Q_1 - a_1) = k(L' - p_1)^2$$

and

$$(Q_2 - a_2) = k(L - L' - p_2)^2$$

The slope of the parabolas at any point is dy/dx, and the parabolas are tangential at B and D.

For parabola AB: $dy/dx = \phi_1 = 2k_1 p_1$

similarly for DE: $dy/dx = \phi_2 = -2k_2 p_2$

As the parabolas are tangential at B and D, the slopes of the two parabolas that meet at each of these points will be equal.

For parabola BCD: $dy/dx = \phi 1 = -2k(L' - p_1)$

and $dy/dx = \phi 2 = 2k(L - L' - p_2)$

Using these equations it is possible to obtain expressions for k_1 and k_2 in terms of k.

$$2k_1p_1 = -2k(L' - p_1)$$
$$k_1 = -k(L' - p_1)/p_1$$

and

$$-2k_2p_2 = 2k(L - L' - p_2)$$
$$k_2 = -k(L - L' - p_2)/p_2$$

Substitute the values of k_1 and k_2 into the original equations for parabolas AB and DE. Therefore:

$$a_1 = kp_1(L' - p_1)$$

and

$$a_2 = kp_2(L - L' - p_2)$$

Substitute the values of a_1 and a_2 into the original equations for parabola BCD. Therefore:

$$Q_1 - kp_1(L' - p_1) = k(L' - p_1)^2$$

and

$$Q_2 - kp_2(L - L' - p_2) = k(L - L' - p_2)^2$$

Solving for k in each case:

$$1/k = [(L' - p_1)^2 + p_1(L' - p_1)]/Q_1$$

and

$$1/k = [(L - L' - p_2)^2 + p_2(L - L' - p_2)]/Q_2$$

These equations rationalise to give the quadratic:

$$jx^2 + mx + n = 0$$

where

$j = (q_1 - q_3)$

$m = (p_2 - 2L)(q_1 - q_2) + p_1(q_3 - q_2)$

$n = (q_1 - q_2)(L - p_2)L$

with the solution $$L' = [-m \pm \sqrt{(m^2 - 4jn)}]/2j$$

Once L' has been calculated, a_1 and a_2 can be found using:

$$a_1 = [(q_1 - q_2)p_1]/L'$$

and

$$a_2 = [(q_3 - q_2)p_2]/[L - L']$$

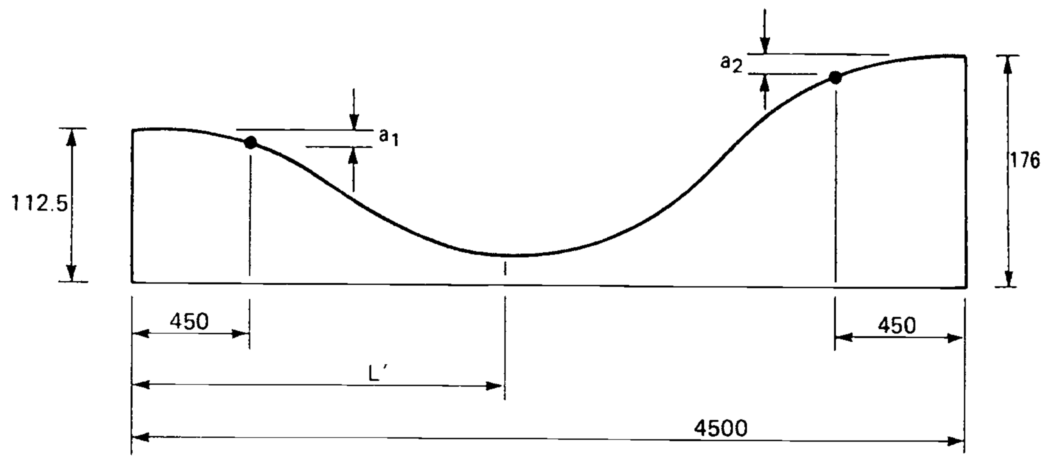


Figure C2: Solution for the transverse direction of Example A1.

For the case shown:

$j \quad = 112.5 - 176 \qquad = \quad -63.5\text{mm}$

$m \quad = [(450 - 2 \times 4500) \times (112.5 - 33)] + [450 \times (176 - 33)] \qquad = \quad -615375$

$n \quad = (112.5 - 33) \times (4500 - 450) \times 4500 \qquad = \quad 1.449 \times 10^9$

$L' \quad = [615375 - \sqrt{}(6153752 + 4 \times 63.5 \times 1.449 \times 10^9)]/(2 \times -63.5) \qquad = \quad 1958.75$

$a_1 \quad = [(112.5 - 33)/1958.7] \times 450 \qquad = \quad 18.27\text{mm}$

$a_2 \quad = [(176 - 33)/(4500 - 1958.75)] \times 450 \qquad = \quad 25.32\text{mm}$

$k \quad = [112.5 - 33]/[(198.62 - 450)^2 + 450 \times (1958.75 - 450)] \qquad = \quad 2.69 \times 10^{-5}$

$k_1 \quad = [-2.69 \times 10^{-5} \times (1958.75 - 450)]/450 \qquad = \quad -9.02 \times 10^{-5}$

$k_2 \quad = [-2.69 \times 10^{-5} \times (4500 - 1958.75 - 450)]/450 \qquad = \quad -12.50 \times 10^{-5}$

APPENDIX D
CALCULATION OF SECONDARY EFFECTS USING EQUIVALENT LOADS

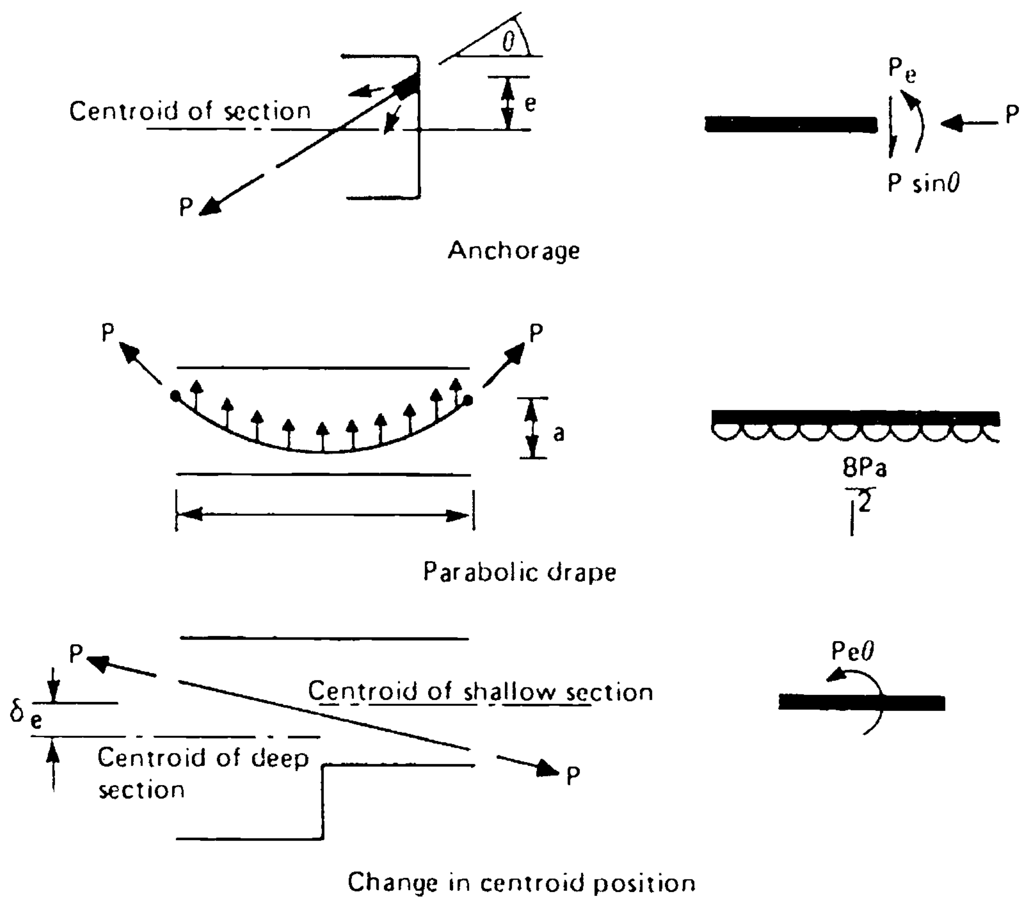


Figure D1: Commonly occurring equivalent loads.

Equivalent loads can be used to represent the forces from prestress. These will automatically generate the combined primary and secondary effects when applied to the structure. Figure D1 shows the commonly occurring equivalent loads for typical prestress situations.

One method of separating the secondary from the primary effects is to use a frame analysis with the equivalent prestress load acting alone. The resultant moment and shear diagrams include both the primary and secondary effects. In order to obtain the secondary effects, it is only necessary to consider the moments and forces at the supports and subtract the primary effects from them. The secondary moments along each span vary linearly from end to end. This method will be known as method A.

To illustrate Method A, the Ultimate Limit State for the transverse direction in Example A1 of Appendix A is used and the secondary effects obtained as follows:

1. Calculate the equivalent prestress loads in the spans using a load factor of 1.0 (Figure D2).
2. Analyse the structure and obtain the bending moment diagram (Figure D3).
3. Calculate the primary moments due to prestress (P_e) in the slab at each support. There are no primary moments in the columns.

At support C	P_e	=	0
At support B(C)	P_e	=	-172/7=24.6kNm
At support B(A)	P_e	=	-172/7=24.6kNm
At support A	P_e	=	0

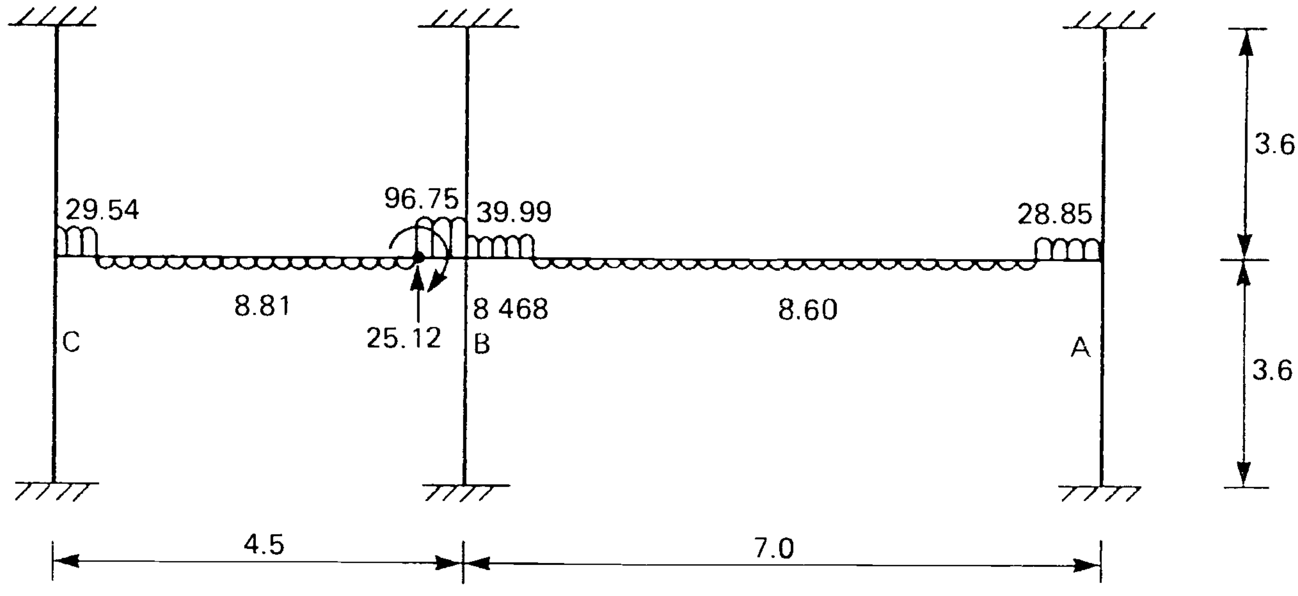


Figure D2: Equivalent balanced loads.

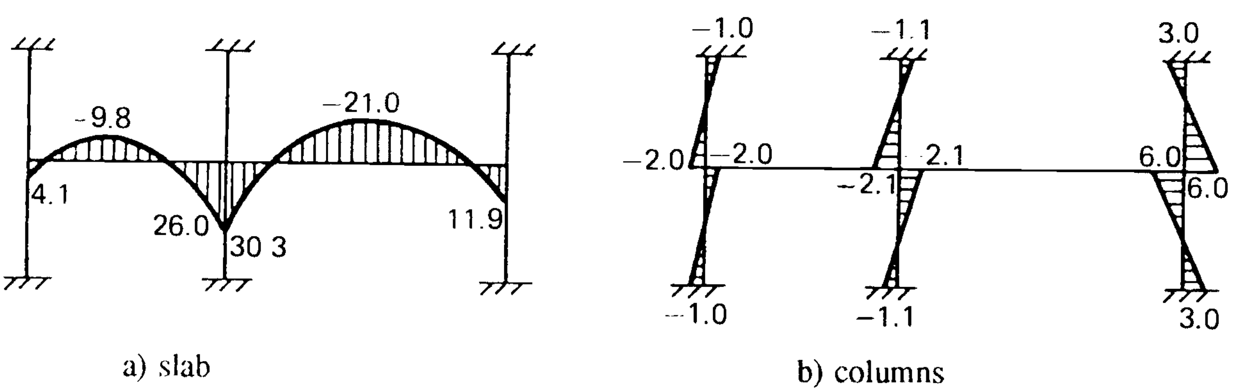


Figure D3: Moments due to primary and secondary effects.

4. Subtract the primary moments from Step 2. At this stage it should be noted that the moments and reactions in the columns from the frame analysis are due entirely to secondary effects (Figures D4 and D5).

An alternative method of calculating secondary effects is detailed below. This will be known as Method B.

As there are no primary prestress forces in the columns, the column moments and reactions are entirely due to secondary effects. So, the secondary effects in the slab can be easily obtained by applying these column reactions and moments to the slab as shown in Figure D6.

This results in the secondary moments and shears in the slab as shown in Figures D4 and D5.

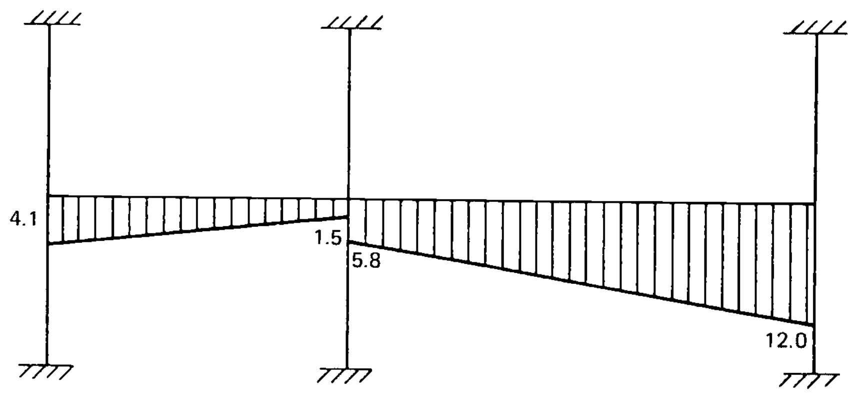


Figure D4: Bending moment diagram due to secondary effects.

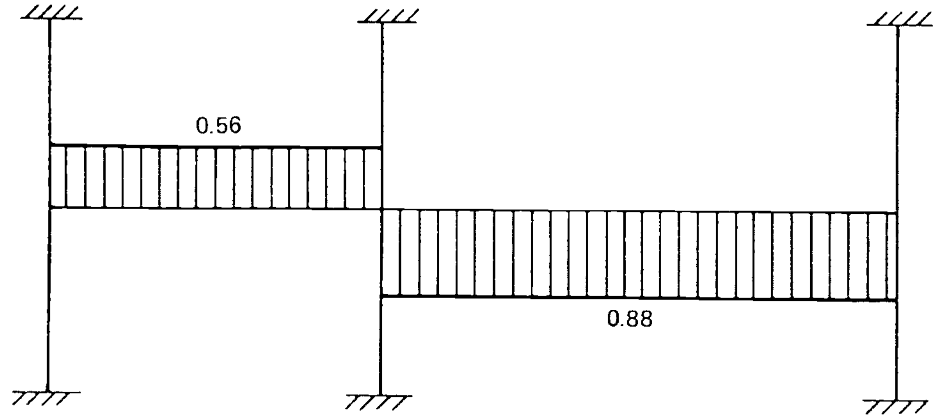


Figure D5: Shear force diagram due to secondary effects.

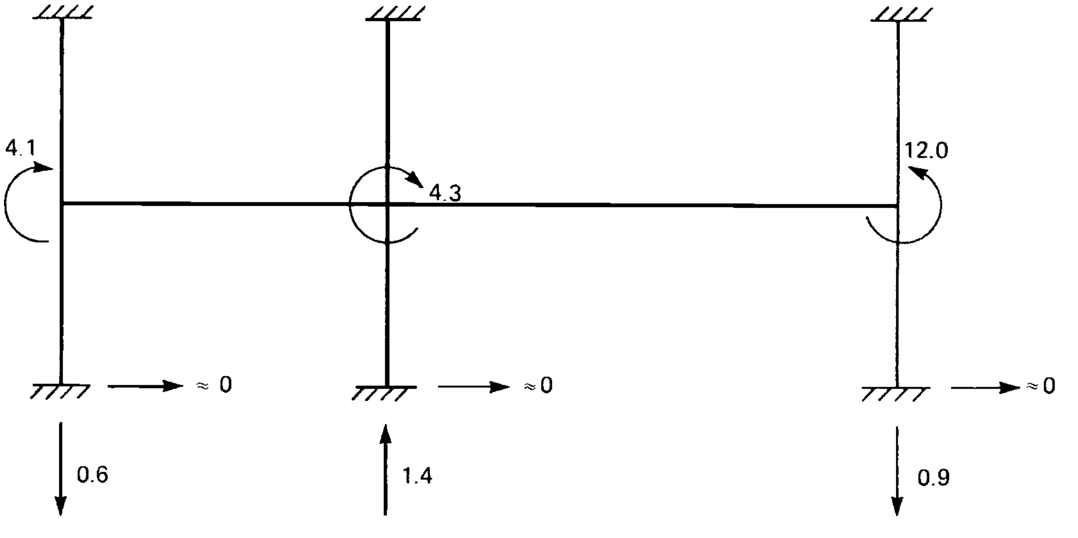


Figure D6: Column reactions and moments due to secondary forces.

APPENDIX E
CALCULATION AND DETAILING OF ANCHORAGE BURSTING REINFORCEMENT

In this appendix two examples requiring bursting reinforcement are considered. For each the un-tensioned reinforcement is calculated and its position in the slab detailed. The first example refers to tendons in Design Example A1 and the second example refers to a broad beam with bonded tendons.

E.1 BURSTING REINFORCEMENT FOR EXAMPLE A1

Reference should be made to Eurocode 2, Clause 6.5.3. Depending on the tendon layout chose from the calculations of Design Example A1 in Appendix A; anchorages will be in groups of 1, 2, 3 or 4. The following example is for a group of tendons 12.9mm strands (unbonded) in a 225mm thick slab, shown in Figure E1.

Limit to steel stress for SLS	=	200MPa
Characteristic strength of tendon	=	186kN

In the y–y direction

Serviceability Limit State

a = 130
b = 225
Jacking force, F = 130.2kN
T = 13.743333kN
(Expression 6.58 of Eurocode 2)
A_s = 68.716667mm^2

Ultimate Limit State

T = 19.633333kN
(Expression 6.58 of Eurocode 2)
A_s = 45.156667mm^2

So use 1 H10 (79mm^2 – c.f. 2 T10 of first version). This un-tensioned reinforcement should be placed between 25mm and 225mm from the front face.

In the x–x direction

Serviceability Limit State

a = 540
h = b: unlimited – say = 1500
F = 520.8kN
T = 97.3896kN
(Expression 6.59 of Eurocode 2)
A_s = 486.948mm^2

Ultimate Limit State

T = 139.128kN
(Expression 6.59 of Eurocode 2)
A_s = 319.9944mm^2

So use 2 × 3H12 (679mm^2 – c.f. 2 × 4T12 of first version). This un-tensioned reinforcement should be placed between 150mm and 1500mm from the front face.

Figure E2 shows the practical detailing of these requirements.

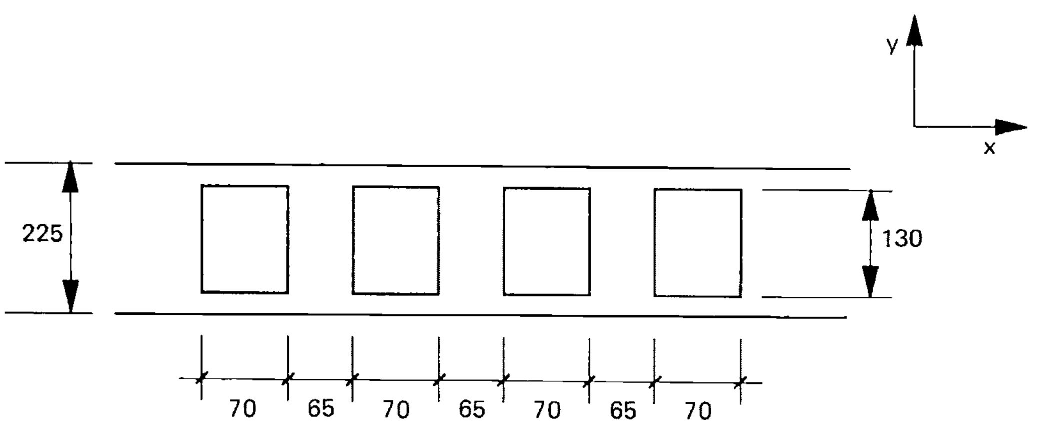


Figure E1: Anchorage layout for Example A1.

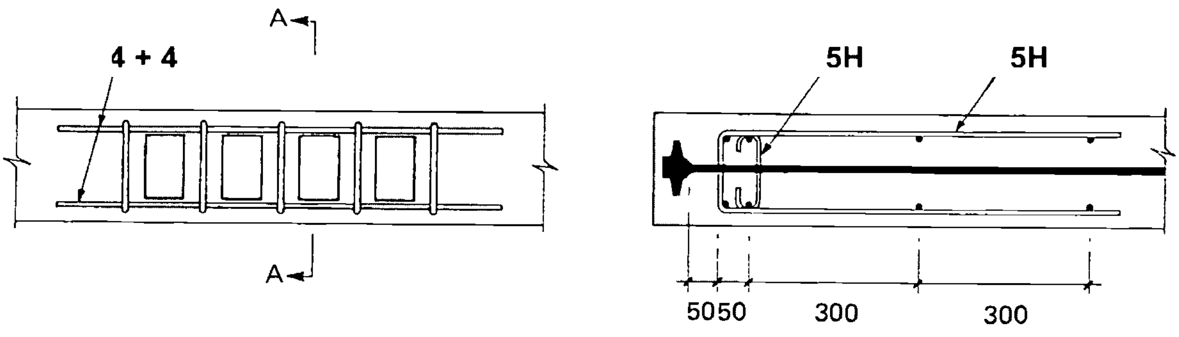


Figure E2: Bursting reinforcement distribution for Example A1.

Comment: It is not usually required to do an equilibrium study for flat plates with regularly spaced tendons, provided they are stressed in such a sequence as to avoid problems at corners.

E.2 BURSTING REINFORCEMENT FOR BROAD BEAM

The design of the anchorage bursting reinforcement for a broad beam with bonded tendons is outlined below. The design requires eleven 15.7mm bonded strands in the beam. It is agreed that two tendons of four strands each and one of three strands are to be used, with anchorages arranged as in Figure E3.

The anchorages are positioned so that the centre of gravity of the tendons corresponds to the centre of gravity of the concrete; this is 196mm above the beam soffit.

Hence, assuming the arrangements in Figure E3,

$$8(\text{strands}) \times (125 + A) + 3(\text{strands}) \times A = 11 \times 196 = 2156$$

Hence $A = 105\text{mm}$

For the stressing sequence, it is assumed that one strand in each tendon is stressed until all strands in all three tendons have been stressed (i.e. if the strands are numbered 1–11 as shown in Figure E3 the stressing sequence would be 1, 5, 9, 4, 8, 11, 2, 6, 10, 3 and 7). In this way, there is no need to consider intermediate stages and it is likely to give the least amount of bursting reinforcement.

In order to check the end block fully, two individual checks are required, namely:

a) single anchorage bursting
b) end block stability.

Figure E4 shows how the end block can be divided into individual end blocks or prisms for each anchorage. These must be rectangular and symmetrical.

Prisms for anchorages 1 and 2 are 125 deep × 750 wide. The prism for anchorage 3 is 125 deep × 1500 wide.

Limit to steel stress for SLS	=	200MPa
Characteristic force per tendon	=	265.5kN
Jacking force per strand	=	185.85kN

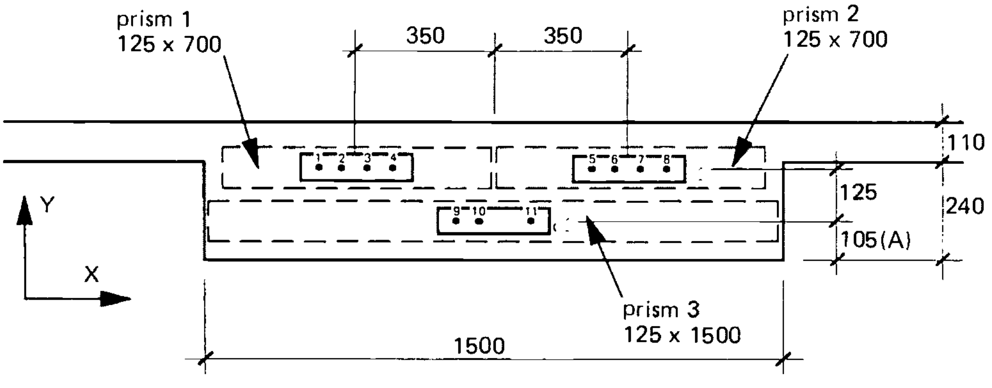


Figure E3: Anchorage layout for Example A1.

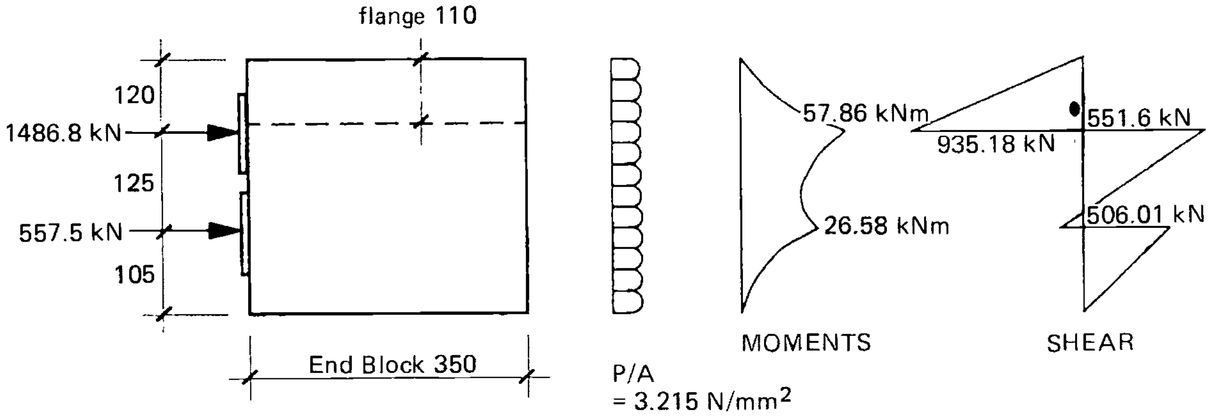


Figure E4: End block moments and forces: y–y direction.

Anchorages 1 and 2

In the x–x direction: Serviceability Limit Stress

a = 275
h = b = 700
F = 743.4kN
T = 134.74125kN (Expression 6.59 of Eurocode 2)
A_s = 673.70625mm^2

This un-tensioned reinforcement should be placed between 70mm and 700mm from the front face.

In the y–y direction: Serviceability Limit State

a = 70
b = 125
F = 743.4kN
T = 81.774kN (Expression 6.58 of Eurocode 2)
A_s = 408.87mm^2

Anchorage 3

In the x–x direction: Serviceability Limit State

a = 275
h = b = 1500
F = 557.55kN
T = 121.49944kN (Expression 6.59 of Eurocode 2)
A_s = 607.49719mm^2

This un-tensioned reinforcement should be placed between 150mm and 1500mm from the front face.

In the y–y direction: Serviceability Limit State

a = 70
b = 125
F = 557.55kN
T = 61.3305kN (Expression 6.58 of Eurocode 2)
A_s = 306.6525mm^2

This un-tensioned reinforcement should be placed between 12.5mm and 125mm from the front face.

Because anchorage forces are increased evenly as explained above, and the anchorages are located on the centre of gravity, the stress block behind the anchorages is uniform and equal to

$11 \times 185.85 \times 103/(0.6359 \times 10^6)$ = 3.215MPa

where (0.6359×10^6) is the area of the section from Example A2.

The moments can be calculated thus:

$M_A = (3.215 \times 1500 \times 2302 \times ½ \times 10^6) - (557.55 \times 125 \times 10^{-3})$ = 57.86kNm
$M_B = 3.215 \times 1500 \times 1052 \times ½ \times 10^6$ = 26.58kNm

Hence, for the maximum moment of 57.68kNm and a lever arm of ½ × block length (= 175mm), steel required

$A_s = 57.86/(0.175 \times 0.200)$ = 1653mm^2

distributed over distance of 175–350mm from the anchorage faces.

From *A guide to the design of anchor blocks for post-tensioned prestressed concrete* (CIRIA)[28], minimum steel

= 0.3% × 1500 × 350 = 1575mm^2, which is OK.

Similarly in the x–x direction.

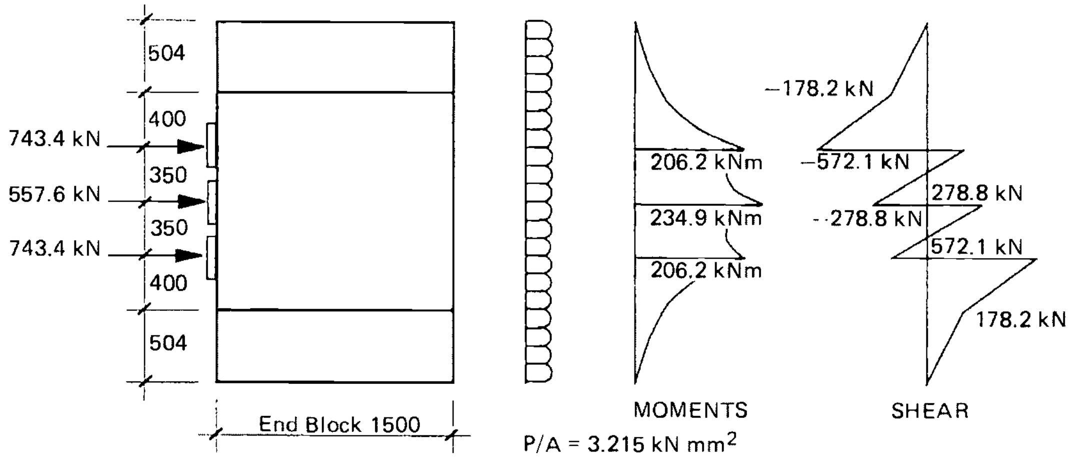


Figure E5: End block moments and forces: x–x direction.

$M_A = M_C$	=	$3.215 \times 110 \times 504 \times (400 + 504/2) \times 10^6$		
		$+ 3.215 \times 350 \times 400^2 \times ½ \times 10^6$	=	206.2kNm
M_B	=	$3.215 \times 110 \times 504 \times (750 + 504/2) \times 10^6$		
		$+ 3.215 \times 350 \times 750^2 \times ½ \times 10^{-6} - 743.4 \times 350 \times 10^3$	=	234.9kNm
Hence,				
A_s	=	$234.9/(0.75 \times 0.200)$	=	1566mm²
Minimum steel	=	$0.3\% \times 350 \times 1500$	=	1575mm²

distributed over distance of 750–1500mm from the anchorage faces.

Note: The above moments are slightly overstated since the anchorage force has been assumed (conservatively) to be a point load.

Flow of stress into flange

Serviceability Limit State

Load in flange = 886.9kN
Width of web = 1500
Effective flange width = 2508mm
a = 1500
h = b = 2508
T = 128.9
A_s = 644.5mm²

This un-tensioned reinforcement should be placed between 250mm and 2500mm from the anchorage faces.

Check on horizontal shear capacity

From Figure E4, maximum shear force	=	935.2kN
giving a shear stress of $935.2 \times 10^3/(1500 \times 350)$	=	1.78MPa
	≤	Shear capacity (see EC2, Exp. (6.21) = $6 \times 113 \times 500/(1.5 \times 75 \times 1500)$ = 2.0MPa, hence OK.

Check on vertical shear capacity

From Figure E5, maximum shear force	=	572.1kN
giving a shear stress of $572.1 \times 10^3/(1500 \times 350)$	=	1.09MPa
	≤	Shear capacity (see EC2, Exp. (6.21) = $3 \times 113 \times 500/(1.5 \times 350 \times 1500)$ = 2.15MPa, hence OK.
In the flange area, maximum shear force	=	178.2kN
giving a shear stress of $178.2 \times 10^3/(1500 \times 110)$	=	1.08MPa
	≤	Shear capacity (see EC2, Exp. (6.21) = $1 \times 113 \times 500/(1.5 \times 110 \times 150)$ = 2.28MPa, hence OK.

The reinforcement layout given in Figure E6 satisfies all the preceding bursting and end-block stability requirements.

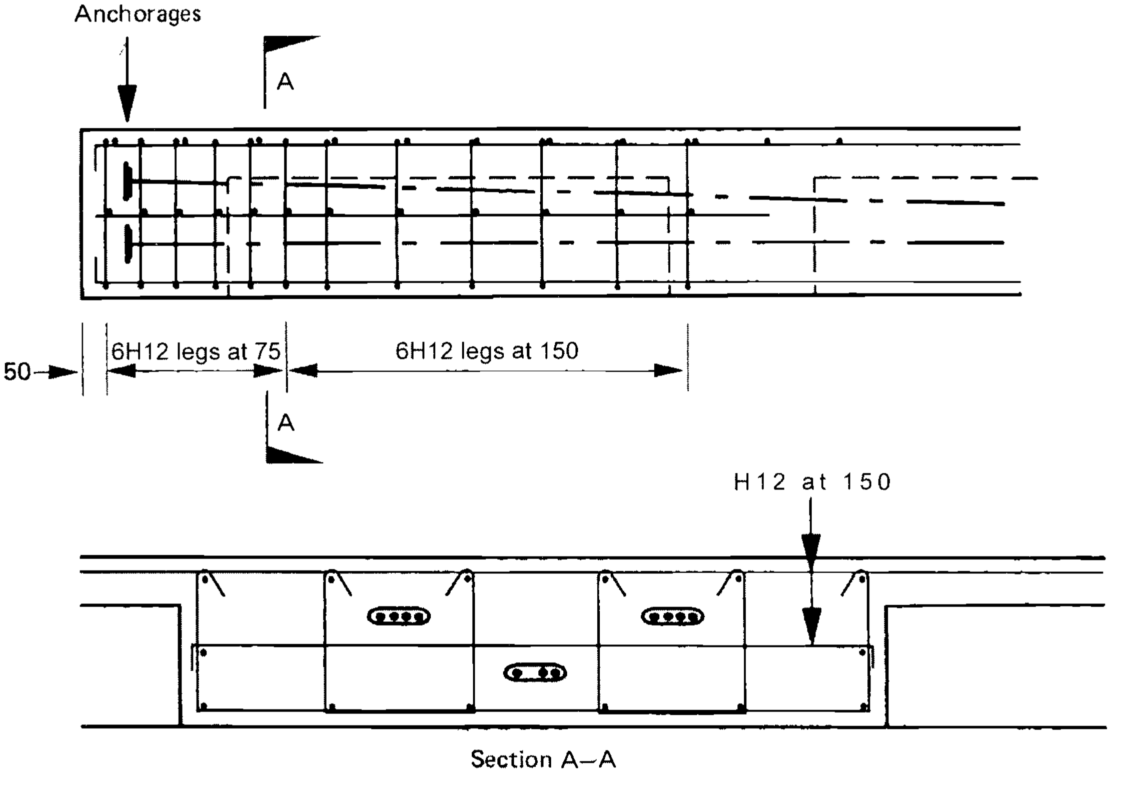


Figure E6: Layout of end block reinforcement.

APPENDIX F
SIMPLIFIED SHEAR CHECK - DERIVATION OF FIGURES 19 AND 20

See Eurocode 2, EN 1992-1-1, Clause 6.4[7].

Assumptions:

1. Charts are drawn for internal columns.
2. $V_c = v_{Rd,c} \times u_1 \times d/1000$ (in kN)

   where
   $v_{Rd,c}$ = shear resistance of the concrete (MPa)
   u_1 = length of the first control perimeter (mm)
   d = equivalent effective depth.

3. $d \approx h - 35$

   where
   h = depth of slab.

4. Columns are square of dimension c
5. $u_1 = 4(c + \pi d) = 4[c + \pi(h - 35)]$
6. Loading is uniformly distributed.

   Ultimate load = 1.42 × (Characteristic dead load + Characteristic total imposed load)

   where
   Characteristic total imposed load, Q_T = Live load + Finishes

7. Concrete density = 24kN/m^3
8. Applied shear force $V = 1.42A\,(24h/1000 + Q_T)$ in kN

   where
   A = appropriate area of floor in m^2

Check at the 1st first control perimeter (Figure 19)

$$V_{Rd} = v_{Rd,c} \times u_1 \times d/1000$$
$$= v_{Rd,c} \times 4(c + \pi(h - 35)) \times (h - 35)/1000$$

$$V \leq V_{Rd}$$

Therefore $1.42A\,(24h/1000 + Q_T) \leq v_{Rd,c} \times 4[c + \pi(h - 35)] \times (h - 35)/1000$

$$\mathbf{Q_T \leq \{v_{Rd,c} \times 4[c + \pi(h - 35)] \times (h - 35)/1000\}/1.42A - 24h/1000kN/m^2}$$

Check at face of column (Figure 20)

Assume f_{ck} = 40MPa

Maximum design shear strength, $v_{Rd,max} = 0.5 \times v \times f_{cd}$
$= 0.5 \times 0.6\,(1 - f_{ck}/250) \times 0.85 \times f_{ck}/1.5$
$= 0.17f_{ck} - 0.00068f_{ck}^2 = 5.71$

$$V_{Rd,max} = v_{Rd,max} \times u_0 \times d/\beta = 19.87cd/1000\text{kN}$$

where
u_0 = $4c$ and $\beta = 1.15$

$$V_{Ed} \leq V_{Rd,max}$$

$$1.42A\,(24h/1000 + Q_T) \leq 19.87cd/1000$$

$$\mathbf{Q_T \leq 14c(h - 35)/1000A - 24h/1000kN/m^2}$$

APPENDIX G
VIBRATION SERVICEABILITY OF POST-TENSIONED CONCRETE FLOORS

G.1 INTRODUCTION

Assessment of floor vibration is an essential serviceability check for modern building structures. The first step in making a reliable assessment is to employ first principles by identifying and characterising the following key factors, taken from ISO 10137[(G1)]:

- the vibration source
- the vibration transmission path (i.e. the mass, stiffness and damping of the floor structure)
- the vibration response and its effect on the vibration receiver.

Historically, two general approaches have been used to assess vibration serviceability of floors: the *frequency tuning method* and the *response calculation method*[(G2)]. The frequency tuning method, based on setting floor natural frequencies above those that can be excited to resonance by the lower harmonic of walking forces was developed first. However, there is now sufficient evidence to show that this method may be unreliable and misleading, and result in uneconomical floor designs. This is particularly so in the case of long-span, heavy and low-frequency floors, such as concrete slabs, where it is difficult and unnecessary to meet typical minimum natural frequencies. Therefore, the frequency tuning method is being replaced in more advanced design guides throughout the world by performance-based methods. In these, the likely vibration response is predicted under the application of realistic dynamic forces. This is the basis of the methods recommended here.

The sources of floor vibration generate dynamic actions, which may vary both in time and in space. They can be divided into two groups, internal and external. External sources, such as traffic and various other types of micro-tremor that excite the whole building, are most efficiently reduced by isolating the whole building or its affected parts, which is beyond the scope of this guide.

This guide deals with vibrations induced by human walking. This is the most important internal source of dynamic excitation of floors accommodating offices, shopping malls, hospitals and other similar types of public buildings and private dwellings. Other special types of floors used in, for example, gymnasia and car parks, may require special considerations of the excitation force and acceptance criteria, which are beyond the scope of this guide.

This guide is written assuming that the reader is familiar with the dynamic behaviour of single and multiple degree-of-freedom (DOF) systems, including linear finite element vibration analysis if required, and with the terminology and concepts of modal analysis and mode superposition techniques[(G3)].

G.2 PRINCIPLES OF FLOOR VIBRATION ANALYSIS

In principle, a methodology for assessing the susceptibility of any floor structure to footfall vibrations should ideally be[(G4)]:

1. Versatile, i.e. applicable to many floor structural forms, no matter how simple or complex they are.
2. Straightforward to use, enabling the consequences of various design iterations to be readily and quickly assessed.
3. Applicable to structures whose dynamic properties may be ascertained by:
   a) *hand calculation*, typically undertaken early in the design process or later in the process when it is required to verify more complex analyses;
   b) *numerical analysis*, typically by a finite element method, in the case of more complex structures; and
   c) *measurement*, typically in cases when a change of usage of an existing floor is proposed, or to aid validation of a complex numerical model.

The response prediction method recommended here satisfies all these requirements. It is based on first principles, and incorporates measured values of footfall forces, employs modelling techniques that predict realistic vibrations, and judges resulting vibration levels against established acceptance criteria. The dynamic response calculations are performed by simple modal analysis and mode superposition techniques[(G3)], so the methodology is versatile. It can be applied to simple regular structures where the modal properties (natural frequencies, mode shapes and modal masses) and dynamic responses can be obtained via readily available formulae and other tabulated data[(G5)], and which is suitable for hand or simple spreadsheet calculations. However, the same methodology for response prediction can also be used on irregular or extensive structures for which finite element analysis needs to be used to obtain reliable modal properties. It should be noted that damping cannot be calculated as such, and always has to be assessed based on experience with floors of similar construction.

Different footfall rates are appropriate for different circumstances. Walking rates above 2.5Hz are uncommon, and this is a reasonable upper limit to the rate for the design of corridors and large circulation areas. For open plan office areas the recommend upper limit is 2.1Hz, and for cellular office areas and laboratories 1.8Hz. However, these more sensitive areas may suffer excessive vibration caused by vigorous walking in adjacent walkway or corridor areas, and this possibility must not be ignored. The methodology proposed has been extensively used and validated both analytically and experimentally over the last ten years.

At the design stage assumptions must be made regarding all the input parameters, some of which have an inherently high variability (e.g. damping, footfall forces). The recommended prediction procedure incorporates design footfall forces higher than average (having a 25% probability of being exceeded), structural modelling techniques and properties that are intended to achieve best estimate modal frequencies and masses, and values of damping that are on the low side of average. While there is still insufficient high quality measured data to give a statistical level of confidence in the whole procedure recommended here, it does predict vibration responses very comparable to those that are measured in practice.

G.3 WALKING EXCITATION

Floors can be divided conveniently into two groups (low-frequency and high-frequency) according to how they respond to walking excitation. Low-frequency floors have modes of vibration that are susceptible to a resonant build-up of vibration under successive footfalls. However, the response of high-frequency floors is not dominated by resonance but by a transient response to the impulsive content of each individual footstep. The natural frequency that separates these two types of response regime is in the region of 10Hz, as described below.

G.3.1 Dynamic load factors for resonant response calculations

The walking forcing function is assumed to be perfectly periodic and presentable by the first four harmonics calculated by Fourier analysis. In reality, dynamic forces from walking are only near-periodic, but for the purpose of analysis they may be assumed to be perfectly periodic. It is assumed that pedestrian-induced resonant response may be possible for floors having natural frequencies up to the frequency of the fourth harmonic of the footfall rate. The fastest normal walking rate f_p does not exceed 2.5 paces per second, that is:

$$f_p \leq 2.5 \text{ Hz} \tag{G1}$$

Therefore, the minimum floor frequency for which resonance can be discounted is approximately 10Hz. Lower values may be appropriate where usage indicates that footfall rates will be lower.

The amplitudes of these harmonics are often expressed in terms of Dynamic Load Factors (DLFs) α_h, which are the magnitudes of the harmonic force components expressed as a fraction of the weight of the walker. Therefore, the harmonic force amplitude P_h of the h^{th} walking harmonic (h = 1, 2, 3 or 4) is:

$$P_h = \alpha_h G \tag{G2}$$

where

G = weight of the pedestrian, usually assumed to be 700N.

There is a considerable scatter in the values of DLFs obtained by various tests; this is illustrated in Figure G1.

Statistical analysis of the data shown in Figure G1 makes it possible to quantify the probabilities that certain force levels will be exceeded. Formulae have been developed for walking rates of up to 2.8 footfalls per second, although rates above 2.5Hz are uncommon in most situations.

Table G1 below shows the proposed mean and design values of the DLFs, with a 25% chance of the design value being exceeded.

G.3.2 Effective impulses for transient response calculations

For floors having natural frequencies above about 10Hz, resonant effects are generally small, and it is more realistic to model footfall loads as a series of force pulses. Appropriate values of an effective impulse I_{eff} have been derived from the same extensive data as that used to calculate the harmonic DLFs. Application of the effective impulse to a mode of given modal mass will predict the same peak vibration velocity of the mode as the footfall time history from which it has been derived. As might be expected, the velocity (and effective impulse) increases with pacing rate and decreases as the natural frequency of the mode increases. If the footfall rate is f_p and the natural frequency is f_n, the proposed effective impulse is shown in Table G2.

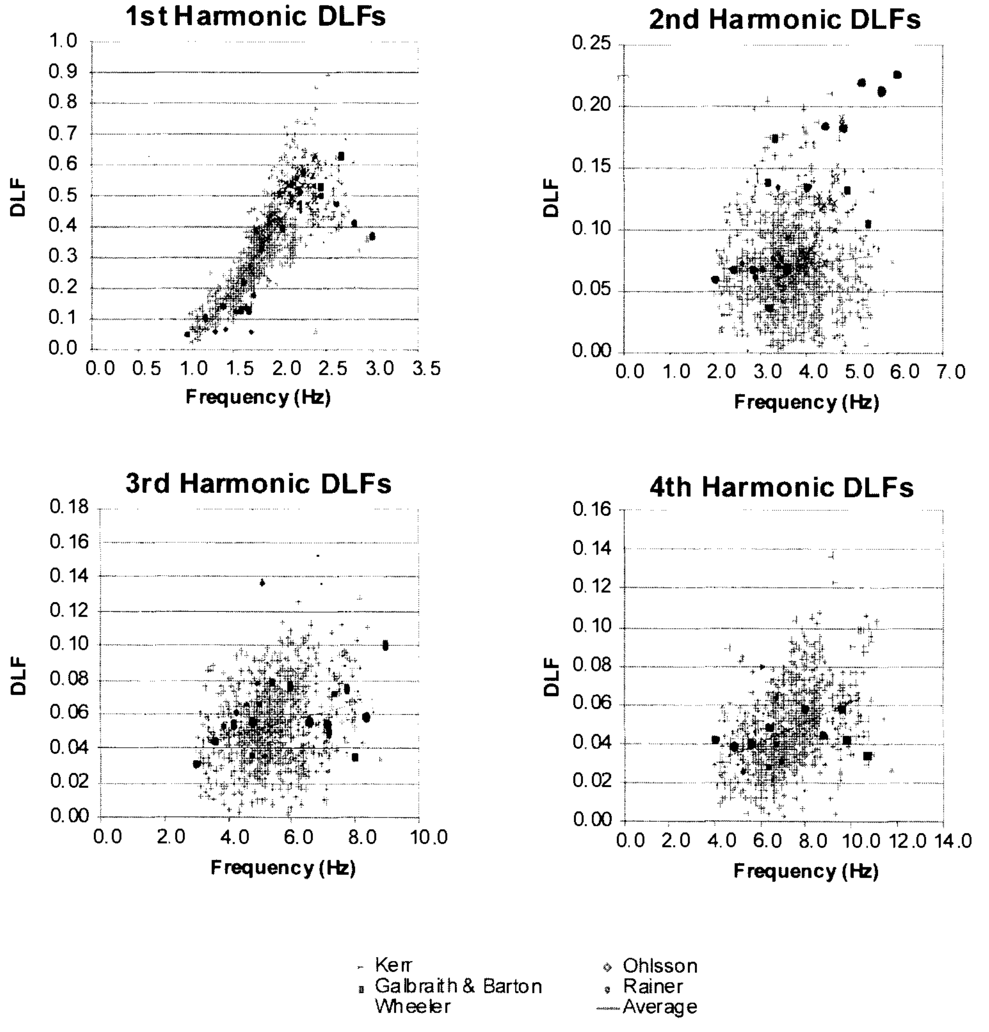


Figure G1: Graphical presentation of the distribution and scatter of DLFs for the first four harmonics of walking, as a function of frequency.

Table G1: DLFs for walking and their associated statistical properties to be used in design.

Harmonic h	**Excitation frequency range for** f_h **[Hz]**	**Mean value of** α_h **as function of harmonic frequency** f_h	**COV of** α_h	**Design value of** α_h **as a function of harmonic frequency** f_h
1	1–2.8	$0.37\ (f_h - 0.95)$*	0.17	$0.41\ (f_h - 0.95)$**
2	2–5.6	$0.0044\ (f_h + 12.3)$	0.40	$0.0056\ (f_h + 12.3)$
3	3–8.4	$0.0050\ (f_h + 5.2)$	0.40	$0.0064\ (f_h + 5.2)$
4	4–11.2	$0.0051\ (f_h + 2.0)$	0.40	$0.0065\ (f_h + 2.0)$

Notes:

COV (Coefficient of variation) is defined as the ratio of standard deviation to the mean value.

* This value is capped to 0.50.

** This value is capped to 0.56.

Table G2: Proposed effective impulse magnitudes.

Mean value of I_{eff} [Ns]	COV of I_{eff}	I_{eff} design value [Ns]
$42\dfrac{f_p^{1.43}}{f_n^{1.30}}$	0.4	$54\dfrac{f_p^{1.43}}{f_n^{1.30}}$

G.4 RESPONSE OF LOW-FREQUENCY FLOORS

The resonant response of low-frequency floors is caused when one or more frequency harmonics of the periodic walking force function are close to a natural frequency of the floor. Having this in mind, the following recommendations apply when calculating response of a low-frequency floor:

1. All modes of vibration having natural frequencies up to 12Hz (1.2 times the cut-off frequency between the low- and high-frequency floors at 10Hz) should be taken into account when calculating the response by mode superposition. This number of modes is denoted as N_m.

2. The steady state acceleration response at a position i in a single mode n of frequency f_n at a given excitation frequency hf_p can be obtained from Equation G3 as follows:

$$a_{i,n}\left(hf_p\right)=\mu_{i,n}\mu_{j,n}\left(\frac{hf_p}{f_n}\right)^2\frac{P_{j,h}}{M_n}\cdot\text{DMF} \qquad \text{(G3)}$$

   Here, hf_p is the harmonic excitation frequency (where f_p is the walking frequency and the harmonic number is h = 1, 2, 3 or 4). The harmonic excitation force of amplitude $P_{j,h}$ is applied at location j (at which the mode shape amplitude is $\mu_{j,n}$). The mode shape amplitude $\mu_{i,n}$ is at location i at which point the response is to be calculated. DMF stands for dynamic magnification factor for steady state harmonic response which, in the case of single mode analysis, is given in Equation G4 as:

$$\text{DMF}=\frac{1}{\sqrt{\left[1-\left(\dfrac{hf_p}{f_n}\right)^2\right]^2+2\zeta_n\left(\dfrac{hf_p}{f_n}\right)^2}} \qquad \text{(G4)}$$

where
ζ_n = viscous damping ratio for mode n.

3. The steady state responses calculated using Equation G3 above will be small for many walking rates, but when the frequency of a harmonic of the footfall rate is close to a natural frequency of the floor, then a larger resonant response will arise at that frequency. If there are several modes with closely spaced natural frequencies, a harmonic force in the region of these frequencies may induce near-resonance in each of these modes. In this case the combined response may be found using the complex number form of the standard steady state harmonic dynamic magnification factor, DMF.

$$\text{DMF}=\frac{1}{\left[1-\left(\dfrac{hf_p}{f_n}\right)^2\right]+i\left[2\zeta_n\left(\dfrac{hf_p}{f_n}\right)\right]} \qquad \text{(G5)}$$

   This is required so that phase information between the contributions from various modes at each harmonic frequency hf_p is maintained as required to calculate the total response at that frequency.

   The total response (at the excitation frequency) is obtained by summing separately the real parts and the imaginary parts of the responses calculated for each of the modes, and then combining the total real and imaginary parts by the SRSS (Square Root of the Sum of the Squares) method[G3] to obtain the overall amplitude of the response at that frequency.

4. For a given walking frequency f_p, once the steady state acceleration responses $a_1(hf_p)$ at each harmonic frequency hf_p have been calculated, the magnitude of the total response due to all harmonics at their corresponding frequencies may be approximated using the SRSS method as follows:

$$a_i=\sqrt{\sum_{h=1}^{4}a_i^2\left(hf_p\right)} \qquad \text{(G6)}$$

5. Two factors will limit the build-up of the resonant response. The worst-case scenario is to assume that the excitation is applied at the anti-node of the mode-shape, and that responses are measured at the same point. If unity-scaled mode shapes[G3] are used, then $\mu_{i,n}=\mu_{j,n}$ = 1.0. However a person who is walking is moving across the structure and is applying forces to different positions along the walking path. Therefore the mode shape value at the excitation point $\mu_{j,n}$ will be different for each footfall. In addition, irrespective of the gradual movement of the loading point, there may be an insufficient number of loading cycles to build to full resonance. This is particularly so in the case of floors with low damping which are excited by the first or second harmonic of walking.

   At resonance $hf_p = f_n$, and Equation G5 gives:

$$\text{DMF}=\frac{-i}{\left[2\zeta_n\left(\dfrac{hf_p}{f_n}\right)\right]} \qquad \text{(G7)}$$

so the imaginary part of the DMF determines the resonant response. To account for these effects, the imaginary part of the DMF may be scaled by a factor r, defined as:

$$r = 1 - e^{-2\pi\zeta_n N} \quad \text{(G8)}$$

where

$$N \approx 0.55h\frac{L}{l} \quad \text{(G9)}$$

In Equations G8 and G9, h is, as before, the harmonic number, L is the span of the floor, and l is the stride length of the individual. The value of r is approximate, but realistic for practical purposes.

6. Using Equations G2 to G9, the total *response factor R* [(G6)] of the floor can be approximated conservatively as:

$$R = \frac{a_{i,RMS}}{a_b} \quad \text{(G10)}$$

In Equation G10, $a_{i,RMS}$ is the calculated root-mean-square (RMS) acceleration whereas a_b is the RMS acceleration in the vertical direction at the threshold of human perception, as defined in BS 6472. Both RMS acceleration levels are usually expressed in m/s^2. Between 4Hz and 8Hz the value of a_b is 0.005 m/s^2. $a_i\,(hf_p)$ are harmonic acceleration amplitudes, so the RMS acceleration at point i can be calculated as:

$$a_{i,RMS} = 0.707a_i \quad \text{(G11)}$$

where a_i is obtained from Equation G6.

G.5 RESPONSE OF HIGH-FREQUENCY FLOORS

Given the fundamental frequency f_1 of a high-frequency floor mode ($f_1 > 10$ Hz), the effective impulse I_{eff} can be calculated using data in Table G2. All modes with natural frequencies up to twice the fundamental frequency should be found and included in the mode superposition calculations. This number of modes is denoted as N_m.

Acceptance criteria in this frequency range are often expressed in terms of velocity. The peak velocity $v_{i,n}$ due to a footfall in each mode may be calculated using:

$$v_{i,n} = \mu_{i,n}\mu_{j,n}\frac{I_{eff,j}}{M_n} \quad \text{(G12)}$$

where all notation is as before and $I_{eff,j}$ is the impulse applied at DOF j.

The total response to each footfall is found by summing in the time domain the decaying transient velocity responses of each mode using the following superposition formula:

$$v_i(t) = \sum_1^{N_m} v_{i,n}(t) = \sum_1^{N_m} \mu_{i,n}\mu_{j,n}\frac{I_{eff,j}}{M_n}e^{-\zeta_n\omega_n t}\sin(\omega_{nd}t) \quad \text{(G13)}$$

where:

$$\omega_{nd} = 2\pi f_n\sqrt{1-\zeta_n} \quad \text{(G14)}$$

and

$$\omega_n = 2\pi f_n \quad \text{(G15)}$$

Equation G13 can be used to estimate peak velocity. However, if required, RMS velocities at DOF i can be calculated using the standard formula:

$$v_{i,RMS} = \sqrt{\frac{1}{T}\int_0^T v_i^2(t)dt} \quad \text{(G16)}$$

where the averaging time T is the worst 1s of largest vibration levels. The calculated peak or RMS velocity can be used to assess vibration serviceability, as appropriate for high frequency floors.

G.6 MODELLING OF MASS, STIFFNESS AND DAMPING OF POST-TENSIONED CONCRETE FLOORS

In the dynamic modelling and analysis of floors for serviceability checks, the following points should be considered[(G7)]:

1. The amplitudes of vibration that arise are generally very small, and it is usual for the structure to act monolithically as if all connections are continuous, even if they are designed as pinned or flexible. Therefore, bending stiffness of the columns can make a considerable contribution to the overall dynamic floor stiffness. As such columns should not be modelled as pin-supports when calculating floor modal properties for vibration serviceability checks. Linear elastic finite element models, where columns are modelled using bar elements rigidly connected to the floor can provide a fairly reliable means of calculating modal properties of in-situ floors.
2. Accurate modelling of the geometry and boundary conditions are of crucial importance when estimating modal properties. Non-structural elements, such as façade walls and partitions can contribute significantly to the stiffness of a floor, and can be modelled if sufficient information about them exists. It is usual for an external façade to provide a line of vertical support along its length.

3. When the floor structure has a different stiffness in the two directions, this should be taken into account. Modelling of this feature using anisotropic shell finite elements with 'smeared' mass and different bending properties in the two directions is reasonable. An alternative approach is to model the slab as a uniform shell and to model the ribs or beams explicitly.
4. Internally, prestressing of concrete elements does not lead to any second-order effects that alter the modal properties.
5. In non-prestressed concrete floors there is usually a degree of cracking under service loads, which can reduce natural frequencies considerably compared with the uncracked condition. The elastic modulus for dynamic analysis of concrete floors is higher than values typically used for structural deflection checks. A value of 38–40GPa is a reasonable assumption in the case of normal strength normal weight concretes. High-strength concrete floors may have increased dynamic modulus of elasticity to about 47GPa. Lightweight concrete has a lower dynamic modulus, in the region of 22GPa.
6. The damping of a floor structure has to be assessed by experience. It is usually expressed as a proportion of *critical damping*, which is the smallest amount of damping that prevents oscillation of an initially disturbed structure. For small strain vibration of bare prestressed and uncracked reinforced concrete structures, the damping ratio is in the region of 0.01–0.02 of critical. The corresponding value for cracked reinforced concrete is slightly higher at 0.015–0.03 of critical. Certain types of fit-out increase the damping, with the most effective improvement arising from full height partitions. Damping in a fully fitted out floor with partitions may reach 0.045 of critical.

G.7 ASSESSMENT OF VIBRATION LEVELS

Vibration in buildings may be deemed unacceptable if it exceeds levels causing adverse human reaction or exceeds values suitable for the operation of sensitive equipment. Assessment of vibration serviceability using these two criteria is discussed below.

G.7.1 Human reaction based on RMS accelerations

BS 6472[G8] forms the basis of guidance on satisfactory levels of vibration for human comfort in the UK. It advises that continuous vibration should be assessed in terms of RMS frequency-weighted acceleration. The acceptability criteria are expressed as multiplying factors on the levels of vibration that are just perceptible. The threshold of perception for continuous vertical vibration is illustrated in Figure G2 (taken from BS 6472) as a function of RMS acceleration versus frequency.

The recommended multiplication response factors R, also taken from BS6472, are given in Table G3.

Footfall-induced vibration depends on a number of factors including walking speed, walking route/path, weight of pedestrian, distance between walker and recipient of vibration, the natural frequency, modal mass and damping of the floor modes of vibration and presence and type of partitions.

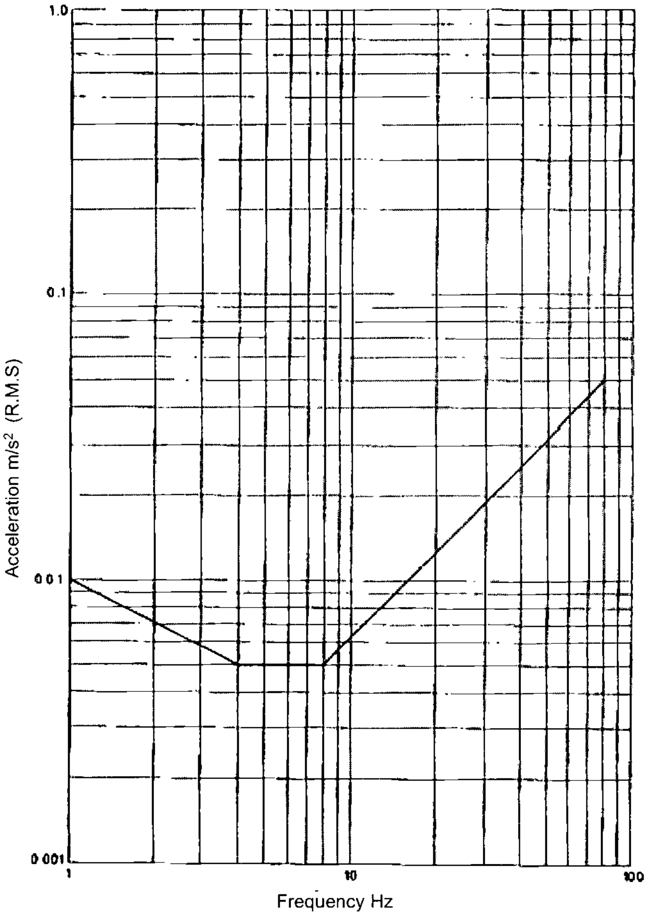


Figure G2: Baseline curve indicating a threshold of perception of vertical vibration.

The very significant variability in the forces produced by different people has been noted previously, and there is also uncertainty in the structural parameters, particularly floor damping.

Also, different people have different vibration perceptibility and acceptability thresholds in any given circumstance. The BS 6472 recommendations are intended to define vibration levels that will lead to a low probability of adverse comment. If vibration levels are twice those recommended then adverse comment may result, and the degree of adverse comment is increased significantly if magnitudes are quadrupled. This illustrates that a noticeable change in human response is associated with significant changes in vibration level, and that small, say 10–20%, changes in vibration levels are insignificant in terms of human reaction.

Table G3: Response factors as proposed in BS 6472.

Place	**Time**	**Multiplying factors (see notes 1 and 5)**	
		Exposure to continuous vibration (16 h day, 8 h night) (see note 2 and Appendix B)	**Impulsive vibration excitation with up to 3 occurrences (see note 8)**
Critical working areas (e.g. hospital operating theatres, precision laboratories (see notes 3 and 10)	Day	1	1
	Night	1	1
Residential	Day	2 to 4 (see note 4)	60 to 90 (see notes 4 and 9, and Appendix B)
	Night	1.4	20
Office	Day	4	128 (see note 6)
	Night	4	128
Workshops	Day	8 (see note 7)	128 (see notes 6 and 7)
	Night	8	128

Note 1: Table 5 leads to magnitudes of vibration below which the probability of adverse comments is low (any acoustical noise caused by structural vibration is not considered).

Note 2: Doubling of the suggested vibration magnitudes may result in adverse comment and this may increase significantly if the magnitudes are quadrupled (where available, dose/response curves may be consulted).

Note 3: Magnitudes of vibration in hospital operating theatres and critical working places pertain to periods of time when operations are in progress or critical work is being performed. At other times magnitudes as high as those for residences are satisfactory provided there is due agreement and warning.

Note 4: Within residential areas people exhibit wide variations of vibration tolerance. Specific values are dependent upon social and cultural factors, psychological attitude and expected degree of intrusion.

Note 5: Vibration is to be measured at the point of entry to the entry to the subject. Where this is not possible then it is essential that transfer functions be evaluated.

Note 6: The magnitudes for vibration in office and workshop areas should not be increased without considering the possibility of significant disruption of working activity.

Note 7: Vibration acting on operators of certain processes such as drop forges or crushers, which vibrate working places, may be in a separate category from the workshop areas considered in Table 3. The vibration magnitudes specified in relevant standards would then apply to the operators of the exciting processes.

Note 8: Appendix C contains guidance on assessment of human response to vibration induced by blasting.

Note 9: When short term works such as piling, demolition and construction give rise to impulsive vibrations it should be borne in mind that undue restriction on vibration levels can significantly prolong these operations and result in greater annoyance. In certain circumstances higher magnitudes can be used.

Note 10: In cases where sensitive equipment or delicate tasks impose more stringent criteria than human comfort, the corresponding more stringent values should be applied. Stipulation of such criteria is outside the scope of this standard.

The basis of checking the acceptability of floors under footfall forces has often been to assess the peak level of vibration and to check this against published criteria based on experience(G7,G9). For normal office floors the multiplying response (R) factor is typically set at 7–8 times the perception threshold. This level is approximately twice the recommendation for offices under continuous vibration given in BS 6472. These R factors corresponding to walking vibrations in offices are based on direct experience of acceptability of footfall-induced floor vibration. They are broadly consistent with BS 6472 on the basis that the maximum footfall-induced response is intermittent rather than continuous as assumed in BS 6472, and is thereby less disturbing. However, it must be noted that when intermittent responses are as high as R=8 then the probability of adverse comment is considerably higher than when R=4, to which a lower probability can be associated. In cases like this, the vibration serviceability check is more an assessment of the risk for adverse comments to be made than a design check with a clear binary pass or fail outcome, common for other types of limit states checks. Clients and their engineers have to get used to this way of thinking about satisfactory vibration serviceability performance.

G7.2 Human reaction based on vibration dose value

In recent years it has been proposed that intermittent vibration should be assessed on the basis of a vibration dose value (VDV):

$$\mathrm{VDV} = \left(\int_0^T a^4 (t) dt \right)^{0.25} \quad \mathrm{m/s^{1.75}} \qquad \text{(G17)}$$

where

$a(t)$= frequency-weighted acceleration

T = total duration of time (in s) during which vibration may occur.

This is a measure of the combined intensity and duration of vibration during a period of time, usually a 16-hour day period or an 8-hour night period. This method is described in detail in Appendix B to BS 6472[(G8)] and is used for the assessment of other intermittent sources such as vibration caused by railway trains. The advantage of the method is that it makes a formal link between vibration intensity, its duration and acceptability which is nowadays accepted to exist. The disadvantage is that a small number of short bursts of strong vibration followed by very quiet periods would be deemed acceptable if VDV is calculated over a long period of time, which may not be the case in all circumstances. While VDV can be calculated using appropriate instrumentation and measured acceleration data, at a design stage it does require the designer to consider what proportions of the time should be assigned to different levels of vibration generated by possible sources.

If the VDV method is used, there is a trade-off between vibration level and duration. Table G4 illustrates the relationship between vibration level and proportion of time such a level needs to exist to generate the same VDV. If vibration is continuous then the proportion of time is 1.0, and the acceptable level is 1.0 times the permissible VDV given in BS 6472, as shown in Table G4 and Figure G3.

If the vibration is intermittent with equal bursts covering 10% of the total time, then the level of that vibration may be 1.8 times the basic permissible level for continuous vibration. Therefore, since BS 6472 proposes a linear relationship between continuous frequency-weighted RMS accelerations and the corresponding VDV, and since the SCI Guide[(G6)] implies that vibration levels of up to R=8 are acceptable for a normal office, whereas BS 6472 recommends R=4 for continuous vibration, it may be deduced that implicit in the SCI Guide is that less than 10% of the time people spend in the office will be affected by the design level of footfall-induced vibration.

For design, it is convenient to calculate footfall-induced vibration in terms of the vibration level caused by typical walk passes. As previously mentioned for high-frequency floors, if calculated vibration time history is available, then the R value is based on the worst 1s of vibration during a walk past. If the number of people crossing the floor each day and night were estimated, together with their walking route, speed and other relevant factors, then a VDV value could be calculated for direct comparison with the recommended limits given in Table G4.

Table G4: Permissible VDV in $\mathrm{m/s^{1.75}}$ *applicable to continuous vibration over 16 or 8 hours, as given in BS6472*[(G8)].

Place	Low probability of adverse comment	Adverse comment possible	Adverse comment probable
Residential buildings 16-hour day	0.2–0.4	0.4–0.8	0.8–1.6
Residential buildings 8-hour night	0.13	0.26	0.51

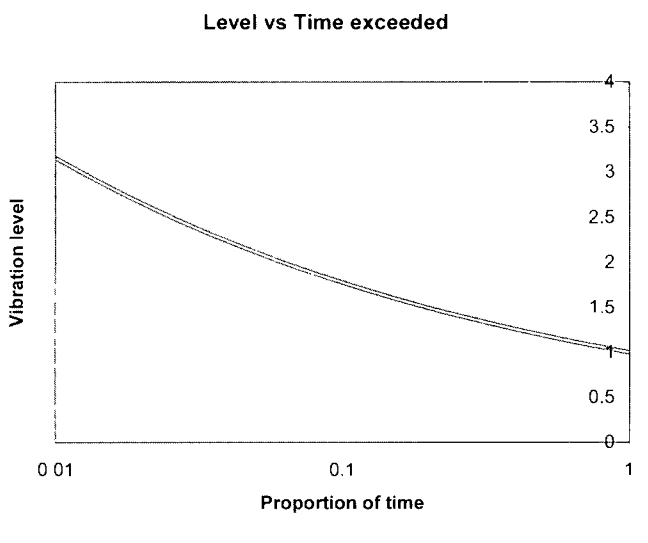


Figure G3: Relationship between a constant VDV and proportion of time and level of actual vibration required to cause such constant VDV.

G.7.3 Effect of vibration on sensitive equipment

Modern medical laboratories and micro-electronic research and production facilities require vibration levels below the threshold of human perception. The many types of equipment have different vibration tolerances, and so an attempt has been made to categorise these and develop generic vibration criteria. Most facilities for these purposes will have stiff high-frequency floors with natural frequencies above 10Hz, and velocity-based criteria are generally specified. The notation of the categories is confusing, with two scales (BBN as given in Reference G10 and ASHRAE[(G11)]) utilising similar-looking lettered criteria, but which are quite different from each other. Table G5 defines the vibration limits (RMS velocity) for a floor to comply with the different generic vibration scales.

Table G5: Generic vibration criteria for equipment[(G12)].

Criterion curve	Maximum velocity Level (see Note 1) µm/sec (RMS)	Detail size (see Note 2) (microns)	Description of use
Workshop (ISO 2631 & BS 6472) ***R=8*** **(see Note 3)**, ASHRAE J	800	N/A	Distinctly perceptible vibration. Appropriate to workshops and non-sensitive areas.
Office (ISO 2631 & BS 6472) ***R=4***, ASHRAE I	400	N/A	Perceptible vibration. Appropriate to offices and non-sensitive areas.
Residential day (ISO 2631 & BS 6472) ***R=2***, ASHRAE H	200	75	Barely perceptible vibration. Appropriate to sleeping areas in most instances. Probably adequate for computer equipment, probe test equipment and low-power (to 20×) microscopes.
Operating theatre (ISO 2631 & BS 6472) ***R=1***, ASHRAE F	100	25	Threshold of perception. Suitable for sensitive sleeping areas. Suitable in most instances for microscopes to 100× and for other equipment of low sensitivity.
VC-A (BBN-A or ASHRAE E) ***R=0.5***	50	8	Adequate in most instances for optical microscopes to 400×, microbalances, optical balances, proximity and projection aligners, etc.
VC-B (BBN-B or ASHRAE D) ***R=0.25***	25	3	An approximate standard for optical microscopes to 1000×, inspection and lithography equipment (including steppers) to 3-micron line widths.
VC-C (BBN-C or ASHRAE C) ***R=0.125***	12.5	1	A good standard for most lithography and inspection equipment to 1-micron detail size.
VC-D (BBN-D or ASHRAE B) ***R=0.0625***	6	0.3	Suitable in most instances for the most demanding equipment including electron microscopes (TEMs and SEMs) and E-beam systems, operating to the limits of their capability.
VC-E (BBN-E or ASHRAE A) ***R=0.03125***	3	0.1	A difficult criterion to achieve in most instances. Assumed to be adequate for the most demanding of sensitive systems including long path, laser-based, small target systems and other systems requiring extraordinary dynamic stability.

Notes:

1. As measured in 1/3 octave bands of frequency over the frequency range 8–100Hz.
2. The detail size refers to the line widths for microelectronics fabrication, the particle (cell) size for medical and pharmaceutical research etc. The values given take into account the observation that the vibration requirements of many items depend upon the detail size of the process.
3. Floor Response Factor R, as defined in SCI Design Guide 076[(G6)].

It should be noted that the RMS in Table G5 is the RMS of a single 1/3 octave frequency band. It is usual for this to be in the region of 70% of the total RMS. This can be checked in detail at the prediction stage by passing the output of the simulated transient response obtained from Section 5 above through digital filters.

APPENDIX H
EFFECT OF EARLY THERMAL SHRINKAGE ON A STRUCTURAL FRAME WITH PRESTRESSED BEAMS

At the time of stressing in a prestressed beam of a concrete frame the two main components of shortening of the beams are elastic shortening and early thermal movement. This example concerns a 90m long post-tensioned beam with six equal spans (see Figure H1). The beam was 575mm deep and 2000mm wide.

No allowance had been made in the design for the effects of shortening movement of the beams at the time of stressing with regard to the interaction with the columns, which at this stage had not been constructed above the beams. The cracking was widespread both in the columns and beams and exceeded 0.7mm in places. Figure H2 shows the different types of cracking that occurred.

CIRIA Report 91[H1], provides a means of calculating the effect for situations of various restraints but does not indicate what value of the restraint factor should be used for such a beam in a structural frame. Furthermore it introduces a modification factor, K, and suggests that this should be taken as 0.5.

The restraint to shortening of the beams by columns was not great for this project. The early thermal movement, including frame action, was calculated to be 8mm. This compared with the free movement of 10mm. However it would have been incorrect to base the movement on the full temperature fall from peak temperature since the beams tried to expand while heating up. A typical curve, showing the temperature rise and fall with time, is shown in Figure H3. This was plotted from temperature measurements of the concrete within a beam on site. The resistance of the columns to the movement of the beams as the temperature rose was more effective than during the cooling phase, since the beam concrete was soft and plastic. It is reasonable to assume that the modification factor of 0.5 simulates approximately the difference between

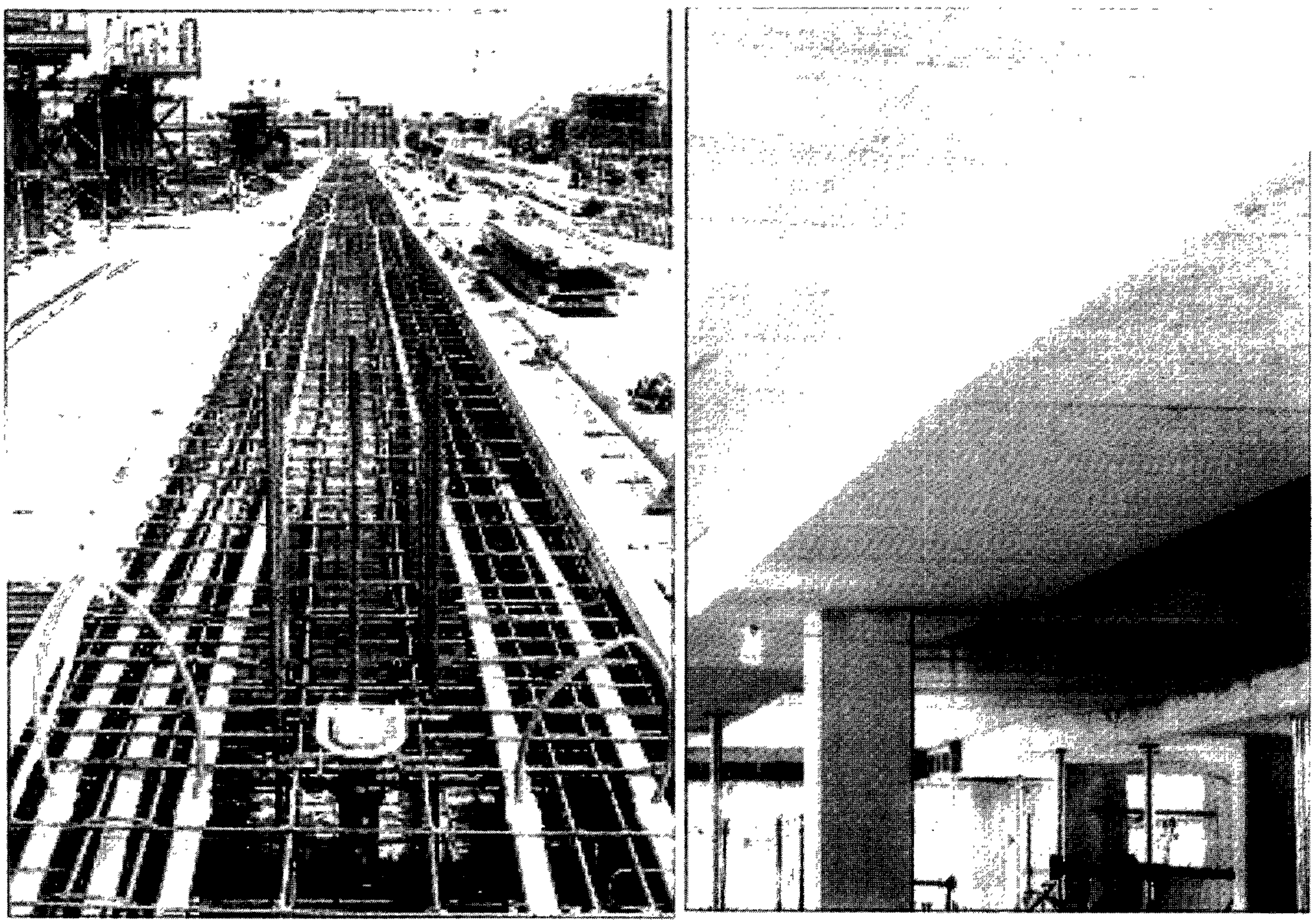


Figure H1: 90m long post-tensioned beam (six equal spans).

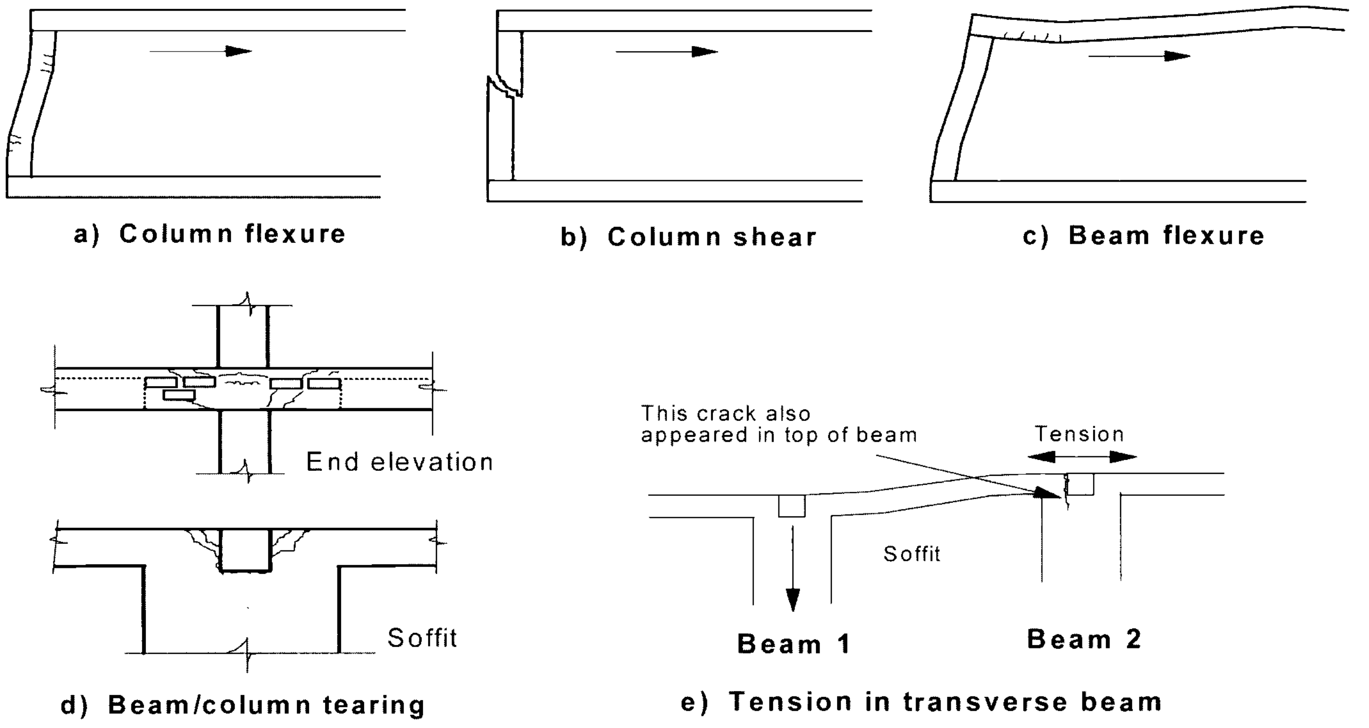


Figure H2: Types of cracking that occurred.

expansion and contraction. Hence for simplicity, the net effect was calculated using half the value for the temperature range with the full coefficient of expansion for hardened concrete (K = 0.5). This gave a reasonably conservative prediction of the actual movement.

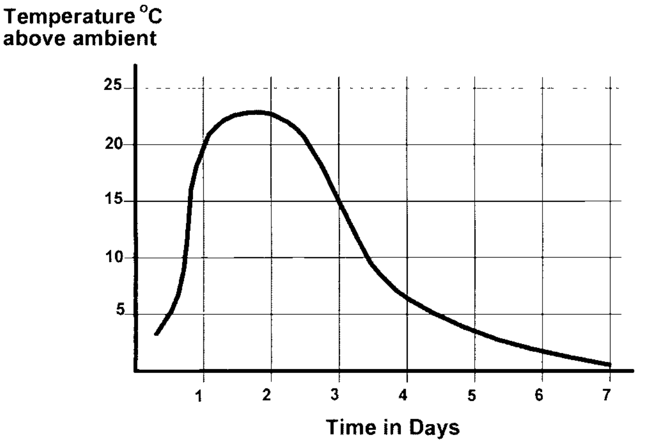


Figure H3: Typical early temperature rise and fall in a concrete beam.

The free early thermal shrinkage of 10mm over the 90mm length of beam corresponds to 111×10^{-6} strain. The free elastic shortening from prestressing at transfer for this beam at this age was 90×10^{-6} strain.

CONCLUSIONS

- For frames with stiff columns early thermal strain is of the same order as the prestress elastic shortening and should be included in the analysis.
- Construction sequence must be considered carefully with respect to shortening effects. Partial prestressing may be necessary.
- Setting-out of the columns should allow for the shortening effects.

Technical Report No. 43

Post-tensioned concrete floors
Design handbook

Since *Post-tensioned concrete floors* was first published in 1994, the use of post-tensioned concrete floors in buildings has continued to grow consistently. Use in the UK is growing rapidly, but their greatest use has been in the USA, especially California, and also in Hong Kong, Australia, Singapore and Europe. Typical applications include offices, car parks, hospitals and industrial buildings.

The first edition of this publication combined various earlier Concrete Society Technical Reports on this subject and expanded some of the recommendations in line with BS 8110. The timely publication of this updated version will update recommendations to the requirements of Eurocode 2 and in light of developments in current practice.

This Report explains the overall concept of post-tensioned concrete floor construction as well as giving detailed design recommendations. The chapters are as follows:

- Introduction
- Structural behaviour
- Structural form
- Materials
- The design process
- Detailing
- Construction details
- Demolition
- Special uses of post-tensioning in building structures
- References

The Appendices to the Report provide valuable additional information. Major worked examples consider the design of post-tensioned flat slabs and the use of finite element analysis, amplifying the approaches given in the main text. Other examples consider the detailed aspects of design, including the calculation of prestress losses, tendon geometry, secondary effects and local bursting reinforcement. Finally, an Appendix deals with the important topic of the vibration behaviour of post-tensioned floors, an area that has not been well covered in the past.

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